OPTIMAL PRODUCTION OF BRA BLOCK FACTORY BY USING SIMPLEX METHOD

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ABSTRACT

In this paper simplex method is used to obtain the optimal solution to maximize the production profit for three different block types in BRA BLOCK FACTORY for heat insulating light construction pomic blocks productions. The three different model that we have taken to investigat the maximum profit for the factory are BB6H.10, BBS.15 and BB2H.20. As aresult we found that the first and the third types of model gives the maximum profit. 

KEYWORDS: Linear Programming Problems; Integer Programming Problems; Simplex Method; Constraints; Objective Function; Variables, Slack Variable; Surplus Variables; Optimal Solution.

1. INTRODUCTION

Simplex method is the first method which was able to solve a linear programming problem developed by the American Air Force during the Second World War. It has proved in practice to perform very well in problems of small or medium size, although (Klee, V., Minty, GJ., 1972) had proven that in the worst case its behavior presents exponential complexity. Since then, a lot of research has been done to find a faster (polynomial) algorithm that can solve LPs. The main branch of this research has been devoted to interior point methods (IPM). There is still much research being done in order to improve pivoting algorithms. Recently, (Paparrizos, 1991) proposed an exterior point Simplex algorithm that avoids the feasible Region; (Luh, H., Tsaih, R., 2002) proposed using IPM to replace phase I of the Simplex; Paparrizos et al. In this research simplex method is used to find optimal solution for the problem that has taken from BRA.BLOCK FACTORY Duhok, three types of block have been chosen of paper B₁, B₂ and B₃ represent the our chosen block types respectively. Furthermore, as it is clear that the linear programing problem is organized in objective function and constraints, in this paper our objective function is the percentage profit in each block types and the constraint part includes five constraints the first two one are mixing and pressing the others constraints are number of our block product in per day.

2. Linear programming problem (Lpp) (Usaha, K., Rashmi, M., 2016), (Bland R., 1977)

A linear programming problem (LPP) is a mathematical program in which the objective function is linear in the unknowns and the constraints consist of linear inequalities. We formulate a mathematical model for general problem of allocating resources to activities. In particular, this model is to select the values for x₁, x₂, ....xₙ as to maximize or minimize

\[ z = c₁x₁ + c₂x₂ + .... + cₙxₙ \]

Subject to constraints:

\[ a₁₁x₁ + a₁₂x₂ + .... + a₁ₙxₙ ≤ b_1 \]
\[ a₂₁x₁ + a₂₂x₂ + .... + a₂ₙxₙ ≤ b₂ \]
\[ .... \]
\[ aₙ₁x₁ + aₙ₂x₂ + .... + aₙₙxₙ ≤ bₙ \]

and

\[ x₁, x₂, .... xₙ ≥ 0 \]

Where:

Z = value of overall measure of performance.

\[ x_j = \text{level of activity ( for } j = 1, 2, ..., n) \]

\[ c_j = \text{increase in Z that would result from unit increase in level of activity } j \]

\[ b_i = \text{amount of resource } i \text{ that is available for allocation} \]
to activities (( for \( i = 1, 2, \ldots, m \))
\[ a_{ij} = \text{amount of resource } I \text{ consumed by each unit of activity } j. \]

3. History of BRA Block factory
BRA plants was established on 02/02/2006 to produce the insulating volcanic light blocks, and the actual operation as on 12/01/2013 to be the largest plant of its kind in the Middle East, the production capacity of up to (100,000 pieces) a day ... and nearly (2,000,000 Piece) monthly using the technology of ADLER French company....

The plant produces more than (25 types) of BRA BLOCK with various dimensions, measurements according to German, European and Turkish standards

3. Computational Procedure of Simplex Method (Dantzig, 1963)
Simplex method by G. Danztig in 1947. The simplex method provides an algorithm (a rule of procedure usually involving repetitive application of a prescribed operation) which is based on the fundamental theorem of linear programming.

This paper aims to find the maximum profit for three types of ponza block which denoted by \( B_1 \), \( B_2 \), and \( B_3 \) of BRA BLOCK FACTORY. Every type needs two stages, mixing and pressing, each step needs a period of time to produce a certain type of block. For this purpose data are collected as shown in table 1 and table 2.
4.1 Types and amount of production for BRA Block

Table (1): Available amount of production per day

<table>
<thead>
<tr>
<th>Block type</th>
<th>Amount available for each day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type (B₁) ((100<em>185</em>390 mm), 4 kg (dry weight), 12.5 pcs./m²)</td>
<td>33000</td>
</tr>
<tr>
<td>Type (B₂) ((150<em>185</em>400 mm), 10 kg (dry weight), 12.5 pcs./m²)</td>
<td>24000</td>
</tr>
<tr>
<td>Type (B₃) ((200<em>185</em>4000 mm), 7 kg (dry weight), 12.5 pcs./m²)</td>
<td>18000</td>
</tr>
</tbody>
</table>

4.2 Duration times of produce

In this stage the block is mixed and pressed in a different period of time as shown in table 2,

Table (2): The Time needs in each stage

<table>
<thead>
<tr>
<th>Types stages</th>
<th>B₁</th>
<th>B₂</th>
<th>B₃</th>
</tr>
</thead>
<tbody>
<tr>
<td>mixing</td>
<td>0.627</td>
<td>0.866</td>
<td>1.155</td>
</tr>
<tr>
<td>pressing</td>
<td>0.133</td>
<td>0.184</td>
<td>0.25</td>
</tr>
<tr>
<td>Available time (per hour)</td>
<td>25200</td>
<td>25200</td>
<td>25200</td>
</tr>
</tbody>
</table>

Now the factory produces 33000 of type B₁, 24000 of type B₂ and 18000 of type B₃ per day. The profit of each product is $0.10, $0.05 and $0.10 respectively.

Table (3): The profit of each types in US Dollar

<table>
<thead>
<tr>
<th>Block Type</th>
<th>Daily Profit</th>
</tr>
</thead>
<tbody>
<tr>
<td>B₁</td>
<td>$3300</td>
</tr>
<tr>
<td>B₂</td>
<td>$1200</td>
</tr>
<tr>
<td>B₃</td>
<td>$1800</td>
</tr>
</tbody>
</table>

5. Formulate the research problem as a linear programming problem

A BRA BLOCK FACTORY produces three types of blocks B₁, B₂ and B₃ and the profit of each product respectively are $0.10, $0.05 and $0.10. Each product is processes in two stages, mixing S₁ and pressing S₂, and needs 99 per second in S₁ stage and 21 per second in S₂ stage. Available time for both stages is not more than 2 minutes. The available times for both stages are not more than 7 hours per day. The limit number of products B₁, B₂ and B₃ respectively are as follows: (330000, 240000, and 180000) per day. In this research we assume that x₁ represents the product number of type B₁, x₂ represents the product number of type B₂ and x₃ represents the product number of type B₃.

Table (4): Formula

<table>
<thead>
<tr>
<th>Stages</th>
<th>Type B₁</th>
<th>Type B₂</th>
<th>Type B₃</th>
<th>Availability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mixing S₁</td>
<td>0.627</td>
<td>0.866</td>
<td>1.155</td>
<td>25200</td>
</tr>
<tr>
<td>Pressing S₂</td>
<td>0.133</td>
<td>0.184</td>
<td>0.250</td>
<td>25200</td>
</tr>
<tr>
<td>Profit</td>
<td>$ 0.10</td>
<td>$ 0.05</td>
<td>$ 0.10</td>
<td></td>
</tr>
</tbody>
</table>

The objective function for this problem is

Maximize \( z_x = 0.10x_1 + 0.05x_2 + 0.10x_3 \)

The constraints are formulated as follows:

\[ 0.627x_1 + 0.866x_2 + 1.155x_3 \leq 25200 \]
0.133x_1 + 0.184x_2 + 0.250x_3 \leq 25200
\quad x_1 \leq 33000
\quad x_2 \leq 24000
\quad x_3 \leq 18000
\quad x_1, x_2, x_3 \geq 0
\hspace{1cm} (5.1)
\begin{align*}
s_1 &= 25200 \\
s_2 &= 25200
\end{align*}

First we start to convert the problem as standard linear programming problem, thus the slack variables should add in each constraint in terms of index (i.e. s_1 in constraint one, s_2 in constraint two and so on)

Maximize \( z = 0.10 x_1 + 0.05x_2 + 0.10 x_3 + 0s_1 + 0s_2 + 0s_3 + 0s_4 + 0s_5 \)

The problem can be solved by using the WinQSB program (Al-safar, 2010).

In this stage the data are tabulated as follow:

**Table (5) : Table of initial data (Simplex Method)**

<table>
<thead>
<tr>
<th>Basic Variables</th>
<th>( C_B )</th>
<th>( X_B )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
<th>( s_5 )</th>
<th>Min ratio ( \frac{X_B}{X_K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_1 )</td>
<td>0</td>
<td>25200</td>
<td>0.627</td>
<td>0.866</td>
<td>1.155</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>40191.38755</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>0</td>
<td>25200</td>
<td>0.133</td>
<td>0.184</td>
<td>0.250</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>189473.68421</td>
</tr>
<tr>
<td>( s_3 )</td>
<td>0</td>
<td>33000</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>33000</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>0</td>
<td>24000</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>18000</td>
</tr>
<tr>
<td>( s_5 )</td>
<td>0</td>
<td>18000</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>18000</td>
</tr>
<tr>
<td>( Z )</td>
<td>( Z=0 )</td>
<td>0.10</td>
<td>0.05</td>
<td>0.10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \Delta_j )</td>
</tr>
</tbody>
</table>

In this step \( s_3 \) is removed and convert to \( x_1 \)

<table>
<thead>
<tr>
<th>Basic Variables</th>
<th>( C_B )</th>
<th>( X_B )</th>
<th>( x_1 )</th>
<th>( x_2 )</th>
<th>( x_3 )</th>
<th>( s_1 )</th>
<th>( s_2 )</th>
<th>( s_3 )</th>
<th>( s_4 )</th>
<th>( s_5 )</th>
<th>Min ratio ( \frac{X_B}{X_K} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( s_1 )</td>
<td>0</td>
<td>4509</td>
<td>0</td>
<td>0</td>
<td>0.866</td>
<td>1.155</td>
<td>1</td>
<td>0</td>
<td>0.627</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( s_2 )</td>
<td>0</td>
<td>20811</td>
<td>0</td>
<td>0</td>
<td>0.184</td>
<td>0.250</td>
<td>0</td>
<td>1</td>
<td>0.133</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( x_1 )</td>
<td>0.10</td>
<td>33000</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( s_4 )</td>
<td>0</td>
<td>24000</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( s_5 )</td>
<td>0</td>
<td>18000</td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>( Z )</td>
<td>( Z=3300 )</td>
<td>0</td>
<td>0.05</td>
<td>0.10</td>
<td>0</td>
<td>0</td>
<td>0.10</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>( \Delta_j )</td>
</tr>
</tbody>
</table>

In this step \( s_1 \) is removed and convert to \( x_3 \)
Finally in above table we get the optimal solution for our problem which is $Z = 3690.38961$ where $x_1 = 33000, x_2 = 0$ and $x_3 = 3903.8961$, here producing 33000 of type B1 and 3903.8961 of type B3 we get $3690.38961$ amount of money as maximum profit.

Case 1
Hence for B1 and B3 we will get a result as the same as above result.

Case 2
If the products of type B2 and product of type B3
Maximize $z_x = 0.05x_2 + 0.10x_3$
Subject to :

$$
0.866x_2 + 1.155x_3 \leq 25200 \\
0.184x_2 + 0.250x_3 \leq 25200 \\
x_2 \leq 24000 \\
x_3 \leq 18000
$$

Using the same technique as used as in solving problem (5.1) the optimal solution $Z= 2054.61893764434$ where $x_2 = 5092.37875288683608$ and $x_3 = 18000$ can be obtained.

Case 3
If the products of type B1 and products of type B2
Maximize $z_x = 0.10x_1 + 0.05x_2$
Subject to :

$$
x_2, x_3 \geq 0 \\
0.627x_1 + 0.866x_2 \leq 25200 \\
0.133x_1 + 0.184x_2 \leq 25200 \\
x_1 \leq 33000 \\
x_2 \leq 24000 \\
x_1, x_2 \geq 0
$$

Using the same technique as used as in solving problem (5.1) the optimal solution $Z = 3560.33487297921$ where $x_1 = 33000$ and $x_2 = 5206.69745958429121$ can be obtained.

### Table (6): The maximum profit for different cases using simplex method

<table>
<thead>
<tr>
<th>Products</th>
<th>Production in seven hours per day</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type B1</td>
<td>33000 0 0 33000 0 33000</td>
</tr>
<tr>
<td>Type B2</td>
<td>0 24000 0 5206.69745958429121 5092.37875288683 0</td>
</tr>
<tr>
<td>Type B3</td>
<td>0 0 18000 0 18000 3903.8961</td>
</tr>
<tr>
<td>Profit $</td>
<td>3300 1200 1800 3560.33487297921 2054.61893764434 3690.38961</td>
</tr>
</tbody>
</table>

Here we need to convert the results to be integer value in constant of rational number because in the real life the above results are not accepted, then the next step we need to use integer programming problem to meet our final aim.

### 6. Integer programming problem (IPP) (Schrijver, 1986)

The linear programming models that have been discussed thus far all have been continuous, in the sense that decision variables are allowed to be fractional. Often this is a realistic assumption. For instance, we might easily produce $102\frac{3}{4}$ gallons of a divisible good such as...
It also might be reasonable to accept a solution giving an hourly production of automobiles at \( 58 \frac{1}{2} \) if the model were based upon average hourly production, and the production had the interpretation of production rates.

At other times, however, fractional solutions are not realistic, and we must consider the optimization problem:

Maximize of objective function:

\[
z = \sum_{j=1}^{n} c_j x_j
\]

Subject to constraints:

\[
\sum_{j=1}^{n} a_{ij} x_j = b_i \quad (i = 1, 2, ..., m)
\]

\[
x_j \geq 0 \quad (j = 1, 2, ..., n)
\]

This problem is called the linear integer programming problem.

It is said to be a mixed integer program when some, but not all, variables are restricted to be integer, and is called a pure integer program when all decision variables must be integers. As we saw in the preceding paper, if the constraints are of a network nature, then an integer solution can be obtained by ignoring the integrality restrictions and solving the resulting linear program. In general, though, variables will be fractional in the linear programming solution, and further measures must be taken to determine the integer programming solution.

First, solve the problem as linear programming problem to obtain optimal solution:

The technique that used in this paper is called Technique rounding solution for solving it. The purpose of using the mentioned technique is obtaining integer number of blocks as it is known that after the calculations sometimes, we get a rational number of objects that should be integer. Therefor in this case we have to change original number to integer because otherwise in real life it does have no meaning.

<table>
<thead>
<tr>
<th>Table (7) : Technique rounding solution</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type B1</td>
</tr>
<tr>
<td>Type B2</td>
</tr>
<tr>
<td>Type B3</td>
</tr>
<tr>
<td>Profit $</td>
</tr>
</tbody>
</table>

**Conclusion**

In conclusion, after defining A BRA BLOCK FACTORY and collecting data concerning of three types of products BB6H.10, BBS.15and BB2H.20, we formulate a problem of three variables representing the block types then used Simplex method and integer programming to find the maximum profit for factory of seven hours per day. As a result, we obtain that producing 33000 of type BB6H.10 and 3903 of type BB2H.20, give the maximum profit. It means that when the factory produces the mentioned number of types BB6H.10 and BB2H.20 it will get $3690.30 as a profit per day.

**References**