Design and FPGA implementation of 11\textsuperscript{th} order Efficient IIR Wavelet Filter Banks with Approximate Linear-phase

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ABSTRACT
In this paper, Bireciprocal Lattice Wave Digital Filters (BLWDFs) are utilized in an approximate linear-phase in pass-band design of 11\textsuperscript{th} order IIR wavelet filter banks (FBs). These filter banks are efficiently designed by replacement one of branches for (BLWDFs) by only a unit delay. The coefficients of the designed filter are achieved by simulating the IIR response suggested in [1]. The design is first simulated using Matlab programming in order to investigate the resulting wavelet filter properties and to find the suitable wordlength for the BLWDFs coefficients. FPGA implementation of the proposed IIR wavelet filter bank is also achieved for three levels with less complexity and high operating frequency.

KEYWORDS: Bireciprocal Lattice Wave Digital Filter (BLWDF), All-pass sections, IIR Wavelet Filter Banks, Approximate Linear-phase, FPGA implementation.

1. INTRODUCTION
Two-channel perfect reconstruction (PR) filter banks have been used in different applications of signal processing, such as subband coding of speech and image signals, transmultiplexers, and voice privacy systems [2]. Filter banks can be realized using finite impulse response (FIR) or infinite impulse response (IIR) filters [3]. Perfect Reconstruction (PR) IIR filter banks are very attractive because of their potentially low system delay and better frequency responses compared with their FIR FBs [4]. H. W. Löllmann and P. Vary [5] in 2008 proposed a new design for a two-channel IIR QMF bank with near-perfect reconstruction (NPR). The analysis filter bank is given by an efficient polyphase network (PPN) implementation based on all-pass filters. These filter banks have a significantly lower algorithmic complexity and causes no or negligible amplitude distortions. This design can be beneficial for coding and transmission applications. In 2009 R. Ramanathan and K. P. Soman [6] presented a novel technique for designing an IIR filter with linear-phase response. This technique uses the frequency domain sampling along with the linear programming concept to achieve a filter design, which gives a best approximation for the linear phase response. The proposed method can give the closest response with less number of samples (only 10) and is computationally simple. Filter design is presented along with its formulation and solving methodology. Numerical results are used to substantiate the efficiency of the proposed method. In 2011 Jassim. M. A. and Rasha. W. H. [7] designed 9\textsuperscript{th} order IIR wavelet filter banks utilizing Bireciprocal Lattice wave Digital Filters (BLWDFs) with approximate linear phase. Each of the two branches in the structure of the BLWDF realizes an all-pass filter. Filter coefficients has been quantized then realized in a multiplierless manner taking into account the low-coefficient sensitivity property of these wave structures. FPGA implementation of such IIR wavelet filter is achieved with less complexity. In 2012, Jassim. M. A. and Sama. N. M. [8] designed a new filter bank structure, for the implementation of discrete wavelet transform. Such structure is based on the idea of the Bireciprocal Lattice Wave Digital Filters (BLWDFs) to simulate the two channel wavelet filter bank for six levels. Linear-phase reconstruction is not treated in such structure. In this paper, 11\textsuperscript{th} order IIR wavelet filter banks are efficiently designed and implemented with perfect reconstruction and approximate linear-phase properties by utilizing bireciprocal lattice wave filter (BLWDF) with the replacement one of its channels by a pure delay unit. The proposed design is then implemented on Spartan-3E FPGA kit. For fast hardware implementation on an FPGA, the filter coefficients are implemented in a multiplierless manner after representing them as sum-and-difference-of-
powers-of-two (SPT). All multiplications are then achieved by shift and add.

2. THE PROPOSED Design

The design of approximately linear phase IIR wavelet filter has been obtained by letting one of the branches \( z^{-1}A(z^2) \) in the BLWDF be a pure delay \( (z^{-2R+1}) \) as shown in Fig. 1. The other branch \( A(z) \) (of even order \( 2N \)) is a general all-pass function in \( z^2 \), which can be realized by cascading all-pass sections [9].

\[
A(z^2) = \frac{z^6 + az^4 + bz^2 + c}{cz^6 + bz^4 + az^2 + 1}
\]

Equation (2) can be written as

\[
H(z) = \frac{1}{2} \left[ c + bz^{-2} + az^{-4} + z^{-5} + z^{-6} + cz^{-9} + cz^{-11} \right]
\]

Also, substituting \( z^{-1} = e^{-j\omega} \) in (4), results

\[
H(e^{j\omega}) = \frac{1}{2} \left[ c + be^{-j2\omega} + e^{-j3\omega} + e^{-j3\omega} + be^{-j4\omega} + c e^{-j11\omega} \right]
\]

The coefficients \( a, b \) and \( c \) are determined by compared the magnitude response for the designed filter with desired magnitude response for \( 11^\text{th} \) order intermediate filter which is given in [1]. The values of \( a, b \) and \( c \) which give minimum error between the two designs (0.0051) are \( a=0.5, b=0.25 \) and \( c=-0.05 \). The magnitude responses for two designs are shown in Fig. 2.

Fig (1) : Approximate linear-phase BLWDF

The transfer function of an approximate linear-phase low-pass BLWDF is

\[
H(z) = \frac{1}{2} \left[ A_0(Z^2) + Z^{-(2R+1)} \right]
\]

where \( R \) is the number of attenuation zeros. For the design of a 11th order IIR wavelet filter of this type, we chose \( N=3 \) and \( R=2 \), then \( A_0(z^2) \) can be written as

\[
A_0(z^2) = z^6 + az^4 + bz^2 + c
\]

and

\[
H(z) = \frac{1}{2} \left[ \frac{z^6 + az^4 + bz^2 + c}{cz^6 + bz^4 + az^2 + 1} + Z^{-5} \right]
\]

Equation (3) can be written as

\[
H(z) = \frac{1}{2} \left[ \frac{c + bz^{-2} + az^{-4} + z^{-5} + z^{-6} + cz^{-9} + cz^{-11}}{cz^6 + bz^4 + az^2 + 1} \right]
\]

The transfer function is

\[
H(z) = \frac{1}{2} \left[ \frac{c + be^{-j2\omega} + e^{-j3\omega} + e^{-j3\omega} + be^{-j4\omega} + c e^{-j11\omega}}{ce^{-j3\omega} + be^{-j4\omega} + e^{-j11\omega}} \right]
\]

The coefficients \( a, b \) and \( c \) are determined by compared the magnitude response for the designed filter with desired magnitude response for \( 11^\text{th} \) order intermediate filter which is given in [1]. The values of \( a, b \) and \( c \) which give minimum error between the two designs (0.0051) are \( a=0.5, b=0.25 \) and \( c=-0.05 \). The magnitude responses for two designs are shown in Fig. 2.
Fig (2) : Magnitude responses of designed filter and desired intermediate filter in [1]

Substituting these coefficients in (4) resulting IIR 11th order BLWDF low-pass transfer function

\[ H(z) = \frac{1}{2} \left[ -0.05 + 0.25z^{-2} + 0.5z^{-4} + z^{-6} + 0.5z^{-7} + 0.25z^{-9} - 0.05z^{-11} \right] \]

The frequency response for the designed filter is shown in Fig. 3.

Fig (3) : Frequency response of 11th order IIR wavelet filter

From Fig. 3, it can be seen that the phase response of the designed filter is almost linear. Linear-phase response can be measured by “deviation from linear phase”. It is defined as a Distance of each point of a phase response of designed filter from the phase response of ideal filter [10]. Fig.4 shows the phase deviation from linear phase of designed filter.

Fig (4) : The phase deviation from linear-phase of the designed filter
for stable and efficient realization, the structure in Fig. 5 is used with $A_0(z)$ rewritten in the following equation [11][12]:

$$A_0(z) = \frac{z^{-3} + az^{-2} + bz^{-1} + c}{cz^{-3} + bz^{-2} + az^{-1} + 1} \quad (7)$$

Fig (5) : An efficient structure for IIR wavelet filter bank

The all-pass function $A_0(z)$ can be expressed in a product form as in 8

$$A_0(z) = \frac{\alpha + z^{-1}}{1 + az^{-1}} * \frac{\beta + z^{-1}}{1 + \beta z^{-1}} * \frac{\gamma + z^{-1}}{1 + \gamma z^{-1}} \quad (8)$$

where the values of $\alpha$, $\beta$ and $\gamma$ are 0.745400849956, 0.16388878629124 and -0.40928965880625, respectively.

The resulting mother wavelet and scaling functions for designed filter shown in Fig. 6 can be generated after five iterations of the analysis filter banks on its low-pass branch.

Fig (6) : The wavelet function and scaling function for the proposed design: (a) wavelet function (b) scaling function

3. OBJECTIVE AND SUBJECTIVE QUALITY METRICS

Different quality metrics exist in practice to evaluate the quality of the signal processing algorithms. Quality measures may be very subjective when based on human perception or can be objectively defined using mathematical or statistical evaluations. The most widely used objective quality metrics are the Signal-to-Noise Ratio (SNR), Peak Signal-to-Noise Ratio (PSNR) and Correlation factor (Cor) [13][14].

4. MATLAB SIMULATION RESULTS

The proposed designed filter bank is described using Matlab programming for verification and selection a sufficient wordlength. Some gray scale images (as shown in Fig. 7) are used to find the minimum wordlength of the BLWDFs coefficients for acceptable PSNR and SNR values which are considered to be about 30dB. The correlation coefficient has a close to one value. The best selected 6-bit wordlength is chosen, which gives the values of the objective quality metrics illustrated in Table (1). These measures show excellent closeness to the ideal case. Thus BLWDF coefficients after quantization are $\alpha = 0.75, \beta = 0.15625$ and $\gamma = -0.40625$. 

doi: 10.25007/ajnu.v7n4a301
Fig (7): The results of Matlab simulation for the 11th order IIR wavelet filter bank on Lena and Barbara images

TABLE (1): The objective quality metrics of the proposed design after quantization

<table>
<thead>
<tr>
<th></th>
<th>Lena</th>
<th>Barbara</th>
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</thead>
<tbody>
<tr>
<td>PSNR (dB)</td>
<td>51.2664</td>
<td>41.2501</td>
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<tr>
<td>SNR (dB)</td>
<td>41.8422</td>
<td>35.3071</td>
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<tr>
<td>Correlation</td>
<td>0.9998</td>
<td>0.9991</td>
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5. FPGA IMPLEMENTATION
The proposed design is described with VHDL language. Then an FPGA device is used to implement the filter design. It is a 500,000-gate Xilinx Spartan-3E XC3S500E in a 320-ball Fine-Pitch Ball Grid Array package (XC3S500EFG320). FPGA implementation is presented for one level and multi-levels. Multi-level implementation is achieved using bit-serial technique which is usually used to reduce implementation cost because of that the hardware resources needed for it are very modest [15]. Bit-serial implementation means constructing a structure that consists of similar cascaded processing elements by building only one of these processing elements and using it to perform the whole task. The implementation results for one level and three-levels are presented in tables 2.

TABLE (2): The implementation results for one level and three-levels

<table>
<thead>
<tr>
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<th>Analysis/ Synthesis</th>
<th>Resource</th>
<th>Used</th>
<th>Available</th>
<th>Utilization ratio</th>
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<td>170</td>
<td>9312</td>
<td>1%</td>
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<tr>
<td></td>
<td></td>
<td>Slice flip flops</td>
<td>163</td>
<td>9312</td>
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<tr>
<td></td>
<td></td>
<td>4-input LUTs</td>
<td>239</td>
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6. CONCLUSIONS
In this paper, a 11th order IIR wavelet filter bank is efficiently designed and realized with approximate linear-phase in pass-band utilizing Bireciprocal Lattice Wave Digital Filters (BLWDFs). The values of the filter coefficients are quite suitable for implementation using shift and add operations instead of multipliers to perform multiplications. This is going to give very efficient hardware saving when implementing, in addition to the method of design with unit delay-all-pass replacement which also gives efficiency in hardware area. The proposed design is first simulated using Matlab programming in order to investigate the resulting wavelet filter properties and to find the suitable wordlength for BLWDF's coefficients. FPGA implementations of one-level and multi-levels of bireciprocal lattice wave digital wavelet filter banks have been obtained with less complexity and high speed.

REFERENCES

<table>
<thead>
<tr>
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<th>Analysis Side Structure</th>
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