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Free Overfall for Discharge Measurement in Channels of Different Shapes: A Review

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ABSTRACT: The end drop in open channels can be considered as a simple and useful tool for the measurement of flow rates flowing in open channels of different shapes. The depth at channel end can be related to the critical depth in the approaching channel and denoted as (EDR). The (EDR) is an essential parameter for measuring end drop discharge (EDD) in open channels. The main target of this paper is to review the previous works done on the end drop in different shapes of channels using the properties of the free end drop flow as basic criteria for the estimation of (EDR) and (EDD). Useful relationships between (EDR) and channel slope are presented which are the outcome results of previous researches done on smooth and rough open channels of different shapes (rectangular, trapezoidal, circular and triangular). Moreover, the correlations of (EDD) depending on end drop depth and characteristics of the upstream approaching channel are presented for channels of different shapes for subcritical and supercritical states of flow. Still there is no unique theoretical solution for the problem of using end depth of free drop as a key for the estimation of flow rates in open channels which needs more theoretical and experimental studies and works on this important field of research.

Keywords: Discharge measurement, free end drop, open channels, different cross sections, subcritical and supercritical flows.

1. Introduction

The problem of providing simple and accurate theoretical and practical approaches of measuring flow rate in open channels at fields and laboratories has been the interesting topic for many hydraulic researchers for at least a century. An open channel with end drop is considered as a measuring discharge hydraulic structure which enables the field engineer to make use of it in irrigation and waste water projects for accurate discharge measurement especially when the amounts of water flow rates are limited and the field owners must pay water consuming invoices and avoid conflicts among them. Analytical computational approaches of measuring flow rates are mostly depending on momentum and energy equations with the consideration of allowable assumptions and some other approaches consider surface profiles as a base for their analytical solutions.

Usually, the water surface profile in a mild sloping channel starts with uniform state of flow after which a gradually varied flow profile prevails until reaching the critical depth section and after that state of flow alters to a rapidly varied profile due to the impact of gravity at the sudden end drop. The flow jet at drop is subject to atmospheric pressure in lower nappe and upper nappe and the flow prevails appreciable curves for the dropping streamlines and eventually ending up with non-hydrostatic pressure distribution at drop and the brink depth becomes smaller than both critical depth and upstream uniform depth. A sketch of a free drop profile in open channel of general cross is shown in Fig. 1.

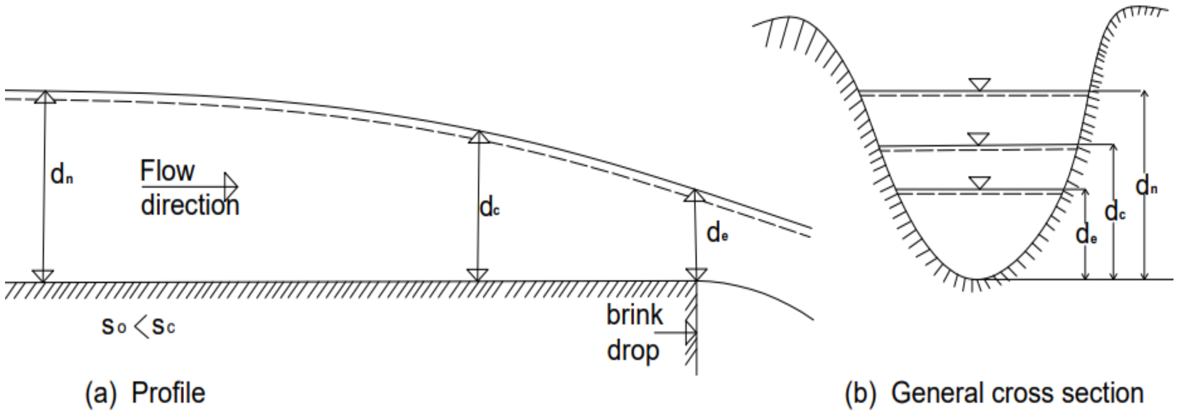

Fig. 1. Free drop profile in open channel of general cross section.

The impacts of channel cross section and other important parameters such as channel bed slope and roughness on establishing (EDR) and (EDD) relationships are presented intensively and analytical approaches and laboratory results of previous investigators are illustrated in a simple way that the hydraulic engineers can apply them to practical problems at field. Moreover, the earlier studies on end depth of free drop carried out by various researches are reviewed in this paper in order to gain a deeper grasp of all its facts. Brief discussions of research papers have been made highlighting key concepts, equations, modeling strategies, results and conclusions.

2. Literature review

The best way of understanding the problem of free end drop in open channels of various cross section shapes is to present the literature review under the headings: channels of rectangular sections, channels of trapezoidal sections, channels of circular sections and channels of triangular sections.

2.1. Channels of rectangular sections

Rouse (1936) perhaps was the primary researcher to see the uncommon qualities of terminal profundity at free overfall for subcritical stream conditions. All the tests were conducted for sloping rectangular channels in which lower nappe and upper nappe at brink were exposed to air concluding that the (EDR) value was found to be 0.715. Rouse (1936) proposed Eq. (1) for the estimation of end depth discharge (Q_R):

$$Q_R = 1.654\ B\ g^{0.5} {d_e}^{1.5} \tag{1}$$

where, g = gravitational acceleration, d_e= brink depth and B = channel bed width.

The data obtained from different bed slopes in rectangular channels have been utilized by Delleur et al. (1956) to investigate the variations in (d_e/d_c) (where, d_c is the critical depth upstream the brink). They discovered that values of the ratio (d_e/d_c) are only dependent on the ratio of bed slope (S_o) to critical slope (S_c) for rectangular channels of rough and smooth beds. Also, they suggested a correlation between the fluctuation of (d_e/d_c) with the relative slope (S_o/S_c) and coefficient of end depth pressure (k_1) ignoring the impact of channel roughness as:

$$\frac{d_e/d_c}{2+k_1(\frac{d_e}{d_c})^3} = \frac{d_n/d_c}{(d_n/d_c)^3} = \frac{(S_c/S_o)^{3/10}}{2+(S_c/S_o)^{9/10}} \tag{2a}$$

where, k_1 = pressure coefficient which is mostly influenced by the relative slope (S_o/S_c) and d_n= uniform normal depth. The (k_1) values were taken as:

k_1= 0.6 for (S_o/S_c) < -5.0 (2b)
k_1= 0.3 + (1- S_o/S_c) / 8 for -5.0 < S_o/S_c < 1 (2c)

k_1= 0.30 for $1 < S_o/S_c$ (2d)

Replogle (1962) performed out his analysis for a rectangular channel using the momentum equation proposed by Diskin (1961) with a number of assumptions. Similar momentum equations were created by him and he showed how little the energy adjustment factor (α), factor for momentum correction (β) and residual force affected the results. He found that for rectangular free fall, the ratios of critical velocity to brink velocity (V_c/V_e) and depth of flow (d_c/d_e) were 0.716 and 1.396, respectively. Replogle (1962) came to the conclusion that the pressure factor was successfully reducing the depth ratio for rectangular overfall. Inaccurate calculations and deviations from the parabolic assumption in the real pressure distribution was the cause of the residual gap between the measured value of (EDR) and the value of 0.715.

Laboratory tests had been undertaken by Rajaratnam and Muralidhar (1964) for smooth channels of rectangular cross sections having various bed slopes in which the results of the tests in sloping channels were found to be mainly dependent on the relative slope (S_o/S_c). It was observed that the results of (EDR) and (EDD) obtained by Rouse (1936), Delleur et al. (1956) and Reploge (1962) for the confined case of rectangular free overfall could be employed for the unconfined rectangular channels as well with acceptable accuracy (little error).

A new technique for calculating end depth ratio has been developed by Anderson (1967) which separately determined and matched the formulas for the area of gravity fall and the channel's water surface profiles. The main outcome of the study was the following equation for the estimation (EDR) for rectangular cross section channels:

$$4(d_e/d_c)^3 - 6(E/d_c)\,(d_e/d_c)^2 + 3 = 0 \qquad (3)$$

where, E = end-section specific energy.

Eq. (3) was simplified for subcritical flow conditions in rectangular channels to the form:

$$4(d_e/d_c)^3 - 9(\frac{d_e}{d_c})^2 + 3 = 0 \qquad (4)$$

Solution of Eq. (4) yields (d_e/d_c) = 0.694. This value gives 3% less than the value of 0.715 obtained by Rouse (1936).

Strelkoff and Moayeri (1970**)** constructed and acquired a boundary value as an integral equation and then conducted it numerically using the potential theory at free drop in rectangular smooth channels. Their results were similar to those of Rouse (1936) and they found that the value of (EDR) for critical states of flow was equal to 0.672 which was significantly lower than the values obtained experimentally.

A laboratory investigation on various roughness kinds in different channel cross sections had been undertaken by Rajaratnam et al. (1976). The study results related to rectangular channels were similar to the results obtained by Delleur et al. (1956) and the study came to the conclusion that the curve of Delleur et al. (1956) was precise for the estimation of (d_e/d_c) for values of the ratio of Nikuradse's sand roughness (k_s) to critical depth (d_c) less than 0.1. For both confined and unconfined rectangular channels, the obtained (EDR) values were 0.715 and 0.705, respectively. With the known values of ($S_o, k_s$and d_c), Rajaratnam et al. (1976) adjusted Delleur's curve for relative roughness to produce the following formulas for rectangular channels:

$$Q_R = 1.654\ B\ g^{0.5} {d_e}^{1.5} \qquad \textit{for confined rectangular channels} \qquad (5)$$

$$Q_R = 1.6893\ B\ g^{0.5} {d_e}^{1.5} \qquad \textit{for unconfined rectangular channels} \qquad (6)$$

Kraijenhoff and Dommerholt (1977) discussed supplementary tests on the free overfall in channels of rectangular sections testing the variations in channel slope and channel roughness. The value of (d_e/d_c) was found to be equal to 0.715 as suggested by Rouse (1936) for values of Reynolds number ranging from 2 x 10^4 to 10^6.

Hager (1983) divided the rectangular channels into two sections, one at the brink and the other farther upstream. The study of the energy and momentum equations between the two sections gave a theoretical foundation for investigating the properties of free drop in rectangular smooth channels. He obtained the formulas shown below to get the ratio (d_e/d_n) for the free drop in horizontal smooth rectangular channels:

$$\frac{d_e}{d_n} = \frac{{F_{rn}}^2}{{F_{rn}}^2 + \frac{4}{9}} \tag{7}$$

where, d_n = upstream uniform normal depth and

$$F_{rn} = \frac{q^2}{g\, {d_n}^3} \tag{8}$$

in which, q = discharge per channel width.

If F_{rn} = 1 in critical state of flow, (d_n) = (d_c) and the answer to Eq. (7) is (d_e/d_c) = 0.6923. Hager (1983) used supercritical flow conditions and high values of F_{rn} to obtain a curve for the fluctuation of (d_e/d_n) and produced the following formula for flow rate (Q_R) as:

$$\frac{{Q_R}^2}{gB^2(d_e)^3} = \frac{2}{2{T_r}^2(1 - T_r)} \tag{9}$$

in which, T_r is calculated as:

$$T_r = \frac{{F_{rn}}^2}{{F_{rn}}^2 + \frac{2}{5}} \tag{10}$$

Ferro (1992) used the discharge-measuring structure of free overfall to investigate the impact of channel width on the variation of (d_e/d_c) in rectangular channels of smooth beds. The study showed that channel width has no practical impact on the relation between brink depth and pressure coefficient. Ferro (1992) presented an equation for the estimation of discharge as:

$$Q_R = 1.51\, B\, g^{1/2} {d_e}^{3/2} \tag{11}$$

(d_e/d_c) ratio was found equal to 0.76.

The issue of end depth for unconfined nappe in channels of rectangular shape and smooth beds was studied by Rai (1993) finding that (d_e/d_c) = 0.712 and (d_e/d_c) was mainly impacted by the ratio (S_o/S_c). The location of critical section was found to lay 3.27 d_c upstream of the horizontal channel's end drop and it depended on the channel bed slope.

Tiwari (1994) employed computer software and the momentum technique to obtain free end drop expression in a channel of rectangular shape and smooth bed. The impact of control volume's weight on a sloping floor was taken into account. For a rectangular channel, he derived the equation:

$$k_1\left(\frac{d_e}{d_c}\right)^3 - 3\left(\frac{d_e}{d_c}\right) + 2 = 0 \tag{12}$$

k_1= pressure coefficient.

(d_e/d_c) value was found equal to 0.66667 in a horizontal channel with zero end pressure which was similar to the value reported by Diskin (1961) in a sloping channel. It was discovered that (d_e/d_c) ratio was influenced by (S_o/S_c) and the results of data analysis were in good agreement with those of the experiments.

Khan and Steffler (1996) presented a numerical model for the sudden drop of flow in vicinity to the end of horizontal rectangular channel having smooth or rough bed and channel having inclined bed with sharp end. Vertically averaged and momentum (VAM) equations were applied to the numerical model using hybrid Petrov-

Galerkin and Bubnov-Galerkin finite elements assuming the pressure and velocity as linear and non-linear distribution. The results predicted from the numerical model for the distributions of pressure and velocity at vicinity of brink agreed quite well with the observed results for horizontal channel bed. Additionally, for weirs of upstream bed slopes between 0.25 and 0.50, the predicted results were in well agreement with those of observed ones, while, for steeper bed slopes the predicted results showed clear divergence from the observed ones.

Davis et al. (1998) experimentally investigated the free end drop for varied slopes and bed roughness in a rectangular channel. The study was carried out in a 0.305 m wide by 3.7 m long metal rectangular flume of plastic sides. The bed was made of painted steel having a slope of 0.0099. Both slope and channel bed roughness were found to have impacts on the ratio (d_e/d_c) and bed roughness having greater impact in case of steeper slopes. Two empirical equations were developed, the second required information on both channel slope (S_o) and roughness (n), whereas, the first just needed data of channel slope only. The equations were:

$$\frac{d_e}{d_c} = 134.84{S_o}^2 - 12.66S_o + 0.778 \tag{13}$$

$$\frac{d_e}{d_c} = 0.846 - 0.219\sqrt{\frac{S_o}{n}} \tag{14}$$

Davis et al. (1998) concluded that the first relationship was more helpful for estimating discharge if bed roughness was unknown.

Ferro (1999) conducted an important theoretical work since it estimates the flow rate using the free drop for channels of different shapes as a function of end depth. This study assumed the pressure distribution as zero and parallel flow lines at end drop, while, ignoring the nappe contraction. He derived the equation shown below to get the flow rate in channels of rectangular shapes and smooth beds (Q_R):

$$Q_R = \mu_R\, B\, g^{1/2} {d_e}^{3/2} \tag{15a}$$

in which, μ_R is defined as:

$$\mu_R = F_{rn}\left[\frac{(2+{F_{rn}}^2)^{3/2} - {F_{rn}}^3}{3F_{rn}}\right]^{3/2} \tag{15b}$$

Where,

F_{rn} = Froude number upstream at the region of uniform depth calculated as:

$$F_{rn} = \frac{V_n}{\sqrt{g d_n}} \qquad for\ subcritical\ flow \tag{15c}$$

$$F_{rn} = \frac{{S_o}^{\frac{1}{2}}\ {R_h}^{\frac{2}{3}}}{n\ g^{\frac{1}{2}}\ {d_n}^{1/2}} \quad for\ supercritical\ \ flow \tag{15d}$$

where, R_h = hydraulic radius, n = Manning roughness coefficient and V_n = upstream average velocity at the region of uniform flow.

Additionally, Ferro (1999) was found the correlation between (F_{rn}) and (F_{re}) hypothesized as (d_e/d_n) ratio:

$$\frac{d_e}{d_n} = \frac{2+{F_{rn}}^2}{2C_s{F_{re}}^2} \tag{16a}$$

where, the distribution factor (C_s) for a rectangular channel cross section was determined as follows:

$$C_s = \frac{(2+{F_{rn}}^2)^{(\frac{5}{2})}-{F_{rn}}^5-2{F_{rn}}^3}{6F_{rn}} - \frac{{F_{rn}}^2}{2}\left[\frac{(2+{F_{rn}}^2)^{(\frac{3}{2})}-{F_{rn}}^3}{3F_{rn}}\right]^3 \tag{16b}$$

Comparison of Ferro's results with the available laboratory results of other researchers showed very well agreements for channels of different shapes.

In order to determine the free fall end depth in channels of rectangular shape having rough beds and steep slopes, Dey (2000) developed a theoretical model based on the momentum approach. The flow profile at free end drops and upstream was solved using a differential equation in expansive channel of rectangular shape taking into account the curvature of streamlines at the free drop. Additionally, the Nikuradse's equivalent sand roughness and discharge at end depth were estimated. The only difference between the obtained results and those of experimental findings was a little more in case of roughness.

The study of Ahmad (2003) for rectangular channel flows of subcritical and supercritical conditions, a quasi-theoretical technique was devised to establish relationships in terms of pressure coefficient (C_{pr}) between end depth discharge and end depth ratio. The main equations produced in this study were:

$$\frac{d_e}{d_n} = \frac{3F_{rn}}{\left[2(1+C_{pr})+{F_{rn}}^2\right]^{3/2} - ({F_{rn}}^2 - 2C_{pr})^{3/2}} \tag{17}$$

$$Q_R = F_{rn}\left[\frac{\left\{2\left(1-C_{pr}\right)+{F_{rn}}^2\right\}^{3/2} - ({F_{rn}}^2 - 2C_{pr})^{3/2}}{3F_{rn}}\right]^{3/2} B\, g^{1/2}\, {d_e}^{3/2} \tag{18}$$

The value of (C_{pr}) was determined using the laboratory findings of (EDR) and (EDD) and projected values were contrasted. The (EDR) value for subcritical flows was 0.758 for unconfined drop and 0.78 for confined drop. It was found that (EDR) decreased with the increase in relative slope (S_o/S_c) and (d_c/B) for subcritical states of flow. Values of (C_{pr}) were found 0.151 and 0.106 for confined and unconfined free falls, respectively. The equations of flow rates in channels of rectangular shape and horizontal bed for subcritical states were found as:

$$Q_R = 1.453\, B g^{1/2} {d_e}^{3/2} \qquad \textit{for confined free drop} \tag{19}$$

$$Q_R = 1.514\, B g^{1/2} {d_e}^{3/2} \qquad \textit{for unconfined free drop} \tag{20}$$

In his experimental study to examine the characteristics of free drop in sloping channels of rectangular shapes and bed roughness, Firat (2004) obtained an expression for flow rate per channel width (q_r) as a function of (S_o), (n) and (d_e) for subcritical and supercritical flow conditions as:

$$q_r = \left(\frac{n}{1.63n + 0.04\sqrt{S_o}}\right)^{3/2} {d_e}^{3/2} \tag{21}$$

Guo (2005) utilized an iterative numerical method depending on the analytical function boundary theory and substitution variables, free rectangular drop in a physical plane is estimated. The estimated values and the experimental results coincided quite well. This method's key advantage was that it was rapid and flexible enough to be used on channel beds of any shape.

Beyrami et al. (2006) studied subcritical state of flow in open channels having different shapes near the border of free drops. A model, theoretically based on the momentum equation and the free vortex theory was employed. End depth ratio (EDR), pressure head distribution, pressure coefficient and flow discharge at the brink were all calculated using the model. The proposed strategy was investigated using available experimental and theoretical findings from other researchers. In rectangular channels, the recommended technique generated values for (EDR) and (K) of 0.7016 and 0.3033, respectively. According to (EDR) results provided by other researchers, the recommended approach resulted in a 1% to 2% difference. With the proposed technique, the deviations between the results of the proposed model and those of other previous formulas ranged between 1.5% and 3%. For the estimation of flow rates (Q_R) in channels of rectangular shape and smooth bed as a function of (d_e), the following equation was presented:

$$Q_R = 1.7016 \ B \ g^{1/2} {d_e}^{3/2} \tag{22}$$

Guo et al. (2008) investigated a rectangular channel with strip roughness for free overfall using both experimental and turbulent numerical models. The channel dimensions were 0.4 m wide, 0.4 m deep and a plywood bed length of 8.4 m. An array of strip roughness measuring 6 mm in height and width fixed transversely on the bed of the channel. A wide range of model parameters including upstream Froude number, channel slope and bed roughness were examined. For different input conditions of velocity in the area between two strips and upstream water surface profile, the results showed that relative roughness spacing had an effective impact on flow condition for a given size of bed roughness and flow condition was reduced with the rise in relative spacing value in this research. Eddies decreased the downstream discharge when the spacing was small but this geometric impact was minimum when the value of relative spacing grew to a particular range.

Tiğrek et al. (2008) tested the relationship between the discharge of a rectangular free fall and the brink depth for smooth and rough channels of rectangular shapes. In an experimental program of testing, different values of Manning coefficient of roughness were tested ranging between 0.0091 and 0.0147 for the range of bed slopes between 0.0003 and 0.0385 and the range of Froude number between 0.41 and 3.68. The values of (d_e/d_c) ratio were estimated using the following equations:

$$\frac{d_e}{d_c} = 0.683 \qquad for\ F_{rn} \le 1 \tag{23}$$

$$\frac{d_e}{d_c} = 0.773 - 0.018\frac{\sqrt{S_o}}{n} \qquad for\ F_{rn} > 1 \tag{24}$$

Additionally, Tiğrek et al. (2008) introduced a discharge coefficient (C_{dr}) and developed the following final discharge relationship for low and high values of Froude numbers:

$$q_R = C_{dr}\ d_e^{\frac{3}{2}} \tag{25a}$$

Where,

$$C_{dr} = \begin{bmatrix} 5.55 & for\ F_{rn} \le 1 \\ \left(\frac{1}{0.361 - 0.00841 S_o^{1/2}/n}\right)^{\frac{3}{2}} & for\ F_{rn} > 1 \end{bmatrix} \tag{25b}$$

where, q_R = flow rate per unit width of the channel.

Mohammed (2009**)** tested three triangular in plan falls of vertex angles (θ) = 60°, 75° and 90° as well as testing a fall making 30° skew with the wall of the channel. He mainly concentrated on the surface profiles of water introducing various articulations correlating the verge profundity with angle (θ) and basic profundity. The release of a 30° skew drop was 21% higher than the release of a straight vertical end drop. The consequences of release prevailed that the flow rate mistake was somewhere in the range of 12% and 15% between determined and estimated releases. Mohammed (2009) investigated the impact of three channel bed slopes (S_o) = 0.0, 0.01 and 0.005 on (d_e/d_c) getting the accompanying relationships as:

$$\frac{d_e}{d_c} = 0.694 - 8.6375\ S_o - 8.253\ {S_o}^2 \quad for\ straight\ vertical\ end\ fall \tag{26}$$

$$\frac{d_e}{d_c} = 0.616 - 4.337\ S_o - 453.23\ {S_o}^2 \qquad for\ 30°\ skew\ end\ fall \tag{27}$$

In an experimental investigation, Mohammed et al. (2011) tried to study the impacts of gravel as bed roughness and slope on channels of rectangular shape with free drop. The investigation was carried out in a 0.3 m wide by 10 m long open channel. Setting the slopes of the channel as 0.0, 0.01 and 0.005, six equations were obtained for the different patterns of gravel of the general form:

$$\frac{d_e}{d_c} = C_1 - C_2 \left(\frac{k\, S_o}{d_c}\right)^{1/2} \tag{28}$$

$C_1$and C_2 are constants dependent on size and patterns of gravel roughness of bed and k = size of gravel.

Mohammed (2012) derived an expression for the determination of flow rate (Q_R) using upstream Froude number (F_{rn}) and end depth (d_e). A triangular plan form drop in a channel of rectangular shape with a free drop was investigated. The discharge coefficient was determined on the presumption that it was equal to the ratio of drop section flow area to uniform normal depth section area. He proposed the following expression for the estimation of discharge (Q_R):

$$Q_R = F_{rn} \left[\frac{(F_{rn}+2)^{3/2} - F_{rn}}{3F_{rn}}\right]^{3/2} (sec\,\theta)^{1/2}\, B\, g^{1/2} {d_e}^{3/2} \tag{29}$$

where, θ = plan form triangle drop's vertex angle.

Mohammed (2012) also calculated a theoretical estimate of (d_e/d_n) as a function of (F_{rn}) of the following form:

$$\frac{d_e}{d_n} = \frac{3F_{rn}\ cos\,\theta}{(F_{rn}+2)^{3/2} - {F_{rn}}^3} \tag{30}$$

Amirabdollahain et al. (2012) used Schwarz-Christoffel transformation and potential flow theory to simulate the free drop flow in channels of rectangular shape. In order to create complex plane of stream function and velocity potential, a complex expression was formulated for transferring horizontal and vertical lines taking into consideration the straight-line segments and operating under the assumption that the flow was invisible and ir-rotational.

Mohammed (2013) investigated how free overfall in a rectangular channel is affected by bed slopes and roughness distribution. Wooden cylindrical elements of 1 cm diameter and 1 cm height were arranged in 2, 3 and zigzag rows and used for the bed roughness arranged in three patterns. Based on the slope of the channel and end drop depth, different flow rate expressions for free drop were developed. For steeper slopes, three rows of bed roughness had a greater impact on these correlations. He also predicted three expressions for (d_e/d_c) and another three for the end depth flow rate.

Hou et al. (2013) used the mesh-less Smoothing Particle Hydrodynamics (SPH) technique to study the free drop in channels of rectangular shape with horizontal and sloping beds. As a result, good agreement was observed between the theoretical and practical results and the projected free surface profiles. In case of horizontal bed scenario, low and high velocity flows were examined. In case of sloping bed issue, high velocity flow rates of high values of Froude number were examined.

Vatankhah (2015) presented a theoretically based method for the determination of (EDR) and (EDD) in channels of different shapes and horizontal beds. This simple method considered the end drop as zero height weir crest in subcritical flows of low Froude numbers. This method was simplified through using common sections such as rectangular, parabolic and triangular. Other researchers' experimental and theoretical findings backed up the proposed relationships. He presented two expressions for ($\frac{d_e}{d_c}$) and (Q_R) as:

$$\frac{d_e}{d_c} = \left[2^{\left(\frac{1}{2}\right)}\, a^{\left(\frac{3}{2}\right)}\, \frac{0.766 + 0.895\, a^{-1.45}}{1 + 0.65\, a^{1.5}}\right]^{-1/a} \tag{31}$$

$$Q_R = a^{-\frac{1}{2}} \left[2^{\frac{1}{2}} a^{\frac{3}{2}} \frac{0.766 + 0.895\, a^{-1.45}}{1 + 0.65\, a^{1.5}} \right]^{\left(1+\frac{1}{2a}\right)} k\, g^{\frac{1}{2}}\, d_e^{\,(a+0.5)} \tag{32}$$

where, k = channel width and a = 1 for channels of rectangular cross sections. For channels with rectangular cross sections, (d_e/d_c) = 0.715.

The morphology of flow nappe after free drop from a channel of rectangular shape having a smooth bed was explored by Zachoval et al. (2013) using few different methods in their study. For a specific comparison situation, the form of the dropping nappe, flow surface profiles before and after the end drop and the circumstances of the water surface and velocity were all specified to get the equation:

$$Q_R = C_{dr}\; g^{1/2}\; B\; d_e^{\,3/2} \tag{33}$$

where, C_{dr}= discharge coefficient, values of C_{dr}were found 1.66 and 1.69 for confined and unconfined free channel drops, respectively for the conditions B > 0.3 m and d_e > 0.04 m.

Swetapadma et al. (2015) carried out a creative experiment on the end drop depth in channels of rectangular shape to study the impact of the main slope in channels on the hydraulics of free overfall. The channel bed was smooth during the experiments and four varied bed slopes (S_o) = 0.0, 0.0024, 0.0035 and 0.0059 were tested during the experimental program. They proposed the following expressions for (EDR) and (EDD) as:

$$\frac{d_e}{d_c} = \emptyset(S_o) = -0.240 * \left(\frac{S_o}{S_c}\right) + 0.725 \tag{34}$$

$$\frac{d_e}{d_c} = 1653 S_o^{\,2} - 31.63 S_o + 0.759 \tag{35}$$

$$Q_R = \sqrt{\frac{g}{(\emptyset(S_o))^3}} * B * d_e^{\,3/2} \tag{36}$$

Maatooq (2017) constructed vertical end drops in a channel of rectangular shape to demonstrate the impacts of channel bed slope and roughness on the end depth ratio. With a total of 72 test runs, three different vales of roughness were used ranging between (n) = 0.009 and 0.019 with different channel slopes (S_o) = 0.0, 0.0025, 0.005 and 0.01. The experimental study concentrated on how (EDR) varied when bed slope and /or roughness changed. Additionally, it had been advised to apply the design formulas as predicted equations for both (EDR) and discharge (EDD) based on statistical derivations. Sand with a (d_{50}) = 0.45 mm and gravel with a (d_{50}) = 4 mm were used as roughness materials. The Manning roughness coefficient of bed (n) was determined as:

$$n = \frac{d_{50}^{\,1/6}}{21.1} \tag{37}$$

Using Eq. (37), values of (n) were determined as 0.013 and 0.019 for sand and gravel, respectively, while, for smooth beds (n) was assumed equal to 0.009. For every bed roughness, an expression was presented for the variation of (EDR) with bed slope. While, keeping channel bed slope constant and changing (n), expressions for the variation of (EDR) with (n) for three bed slopes were proposed. For the combined impacts of both (S_o) and (n) on the (EDR), Maatooq (2017) proposed the following equation:

$$\frac{d_e}{d_c} = 0.6387\left(\frac{\sqrt{S_o}}{n}\right)^{0.0714} \tag{38}$$

Moreover, an expression for the dimensionless unit discharge ($q_R/(\sqrt{g} d_e^{\,1.5})$) as a function of (EDR) was presented as:

$$\frac{q_R}{\sqrt{g}{d_e}^{1.5}} = \frac{1}{(\frac{d_e}{d_c})^{1.5}} \tag{39}$$

in which, q_R= discharge per unit width.

Finally, Maatooq (2017) proposed a relationship between (${S_o}^{\frac{1}{2}}/n$) and (d_e/B) of the form:

$$\frac{{S_o}^{\frac{1}{2}}}{n} = \frac{2.13}{(d_n/B)^{1.338}} \tag{40}$$

Swetapadma et al. (2014) and Nabavi (2015) analyzed the correlations between the end drop depth and critical depth for the free drops in channels of rectangular shapes. The majority of studies conducted until 2015 were reviewed. For flow in rough channels of subcritical, critical and supercritical conditions, substantial relationships between brink depth, channel bed slope and bed roughness were also highlighted in both investigations. They showed the (EDR) values for each researcher in accordance with the parameters in the experimental or theoretical investigation illuminating the case of the overfall whether it is confined or unconfined.

Very useful theoretical approach for the computation of EDR for channels of various end drop cross sections was well proposed by Abrari et al. (2019) for subcritical flow conditions through combining both energy and continuity equations for their solution. The fruitful outcome of the approach was the following equation for the (EDD) in rectangular free overfall channels:

$$Q_R = 1.682 * B * g^{\frac{1}{2}} * {d_e}^{\frac{3}{2}} \tag{41}$$

(EDR) = 0.707 for rectangular shape channels.

2.2. Channels of Trapezoidal Sections

Diskin(1961) derived an equation employing the momentum principle for calculating brink depth in channels of trapezoidal and exponential cross section shapes assuming the pressure at brink to be zero for horizontal trapezoidal channels as:

$$(X_c + (X_c)^2)/(X_e + (X_e)^2) = (10{X_c}^2 + 20X_c + 9)/[6(1 + {X_c}^2)] \tag{42}$$

where, $X_c = z\, d_c/B$, $X_e = z\, d_e/B$ and z = channel side slopes.

Rajaratnam et al. (1962) derived an equation for trapezoidal channels assuming non-zero pressure at brink depended on the momentum principle of the form:

$${X_e}^5 + {X_e}^4 + {X_e}^3 - [(\varphi_1(X_c) + 1)/(K_1\varphi_2(X_c))]{X_e}^2 - [(\varphi_1(X_c) + 1)/(K_1\varphi_2(X_c))]X_e + [(\varphi_3(X_c))/(K_1\varphi_2(X_c))] = 0 \tag{43a}$$

in which,

$$\varphi_1(X_c) = (3 + 2(X_c))/[6(1 + (X_c))] \tag{43b}$$

$$\varphi_2(X_c) = (3 + 2(X_c)/[(Xc + {X_c}^2)\ \varphi(1 + X_c)X_c] \tag{43c}$$

$$\varphi_3(X_c) = X_c + {X_c}^2 \tag{43d}$$

Moreover, Rajaratnam et al. (1962) suggested several modifications to Diskin's (1961) approach.

Rajaratnam and Muralidhar (1970) conducted experimental investigation on smooth horizontal trapezoidal free overfall channels in addition to the analysis of Diskin's (1961) data to get the following functional expression for (EDR):

$$\frac{d_e}{d_c} = f(\frac{S_o}{S_c}, z\, d_c/B) \tag{44}$$

Subramanya and Murthy (1987) solved the problem of smooth channels of trapezoidal free drop shapes with horizontal beds using the consideration of energy and water surface profile continuity at end drop of the channel to obtain an equation for the brink depth for subcritical flow conditions as:

$$\epsilon(\Phi) - 4\eta - 3\, f(\varphi, \eta) = 0 \tag{45a}$$

in which,

$$\epsilon(\Phi) = 1 + 1/2\, [\frac{1+(\varphi)}{1+2(\varphi)}] \tag{45b}$$

and

$$f(\varphi, \eta) = (1+\varphi)^3/[(1+\ \varphi\eta)^2\eta^2(1+2\varphi)] \tag{45c}$$

The results of the above equations showed deviations of $\pm 2\%$ compared with the experimentally obtained data.

Keller and Fong (1989) used momentum principle and assumed that the pressure at brink is not zero to solve the trapezoidal channel equation. In addition, they investigated the trapezoidal overfall through experimentation. The experimental data were compared to the predicted relation between end depth and flow rate. For maintaining a subcritical flow state during the whole testing program, the experiments of the study were carried out in a 3 m long channel with a 0.15 m base width, 1V:1H side slopes and a 0.00067 slope of the channel bed. According to bed width (B) and side slopes (z), the theoretical study's change in brink depth (d_e) with critical depth (d_c) was stated as the following sixth degree expression:

$$10\, {X_c}^4 + 20\, {X_c}^3 + 9\ {X_c}^2 - \frac{6}{G_1}(\, {X_c}^3 + 3\, {X_c}^4 + 3\, {X_c}^5 +\ {X_c}^6) - KG_1G_2(1+2X_c) = 0 \tag{46a}$$

in which,

$$G_1 = X_e + {X_e}^2 \tag{46b}$$

$$G_2 = \frac{(3+2X_e)X_e}{1+X_e} \tag{46c}$$

where, $X_e = \frac{z\, d_e}{B}$, $X_c = \frac{z\, d_c}{B}$ and K = pressure factor.

The pressure factor was evaluated as K = 0.215 for trapezoidal channels and K = 0.175 for ninety-degree triangular channels based on the measurements of Reploge (1962).

Gupta et al. (1993) conducted research on smooth trapezoidal free overfall channel with different bed slopes. Using dimensionless parameters, a calibration curve was presented. To predict the flow rate (Q_{Tr}) as a function of (d_e) , Gupta et al. (1993) used the non-dimensional terms [$Q_{Tr}\, z^{1.5}/\sqrt{gB^{2.5}}$] and [$e^{5.5(S_o)}z\, d_e/B$]. The curve that most closely matched the data had a coefficient of correlation = 0.99753. Additionally, it was noted for horizontal channels that the slope value in the equation of the straight-line relation between (X_e) and (X_c) was higher than that observed from the actual data. Gupta et al. (1993) presented an expression for (Q_{Tr}) in terms of (d_e) as:

$$x = a\, y^2 + b\, y + c \qquad for\ \ -1.45 < y < 0.95 \tag{47a}$$

in which, a = 0.2217865, b = 1.959574, c = 0.4874965, and

$$x = \ log_{10}\left[\frac{Q_{Tr}\ z^{1.5}}{g^{\frac{1}{2}}\, B^{2.5}}\right] \tag{47b}$$

$$y = log_{10}\left[e^{(5.5\,S_o)}\,\frac{z\,d_e}{B}\right] \tag{47c}$$

Also, Gupta et al. (1993) observed that (d_e/d_c) value for horizontal and positive slopes of trapezoidal channels were 0.745 and 0.726, respectively.

Tiwari (1994) produced a computer software based on the momentum representation for channels of trapezoidal shape sections with free drops. This approach took into account the weight of the control volume's impact on the slope of the floor. He came up with an equation for a horizontal trapezoidal channel with K = 0, bed width of and side slopes that was identical to Diskin's equation. It was also shown that (d_e/d_c) ratio was dependent on relative slope for channels having bed slopes.

A general flow rate expression for the free end drop in channels of trapezoidal cross section shapes was theoretically proposed by Anastasiadou-Partheniou and Hatzigiannakis (1995) via modelling the flow lines over-passing the end drop as flow over-passing a sharp weir and considering the impact of curvature of stream lines at drop. A general expression for plotting water surface profiles was also proposed. The equations depicted below were advised to be used for the determination of (Q_{Tr}) for low and high flow velocities:

$$Q_{Tr} = \frac{F_{rc}\,\sqrt{g}\,B^{\left(\frac{5}{2}\right)}}{z^{\left(\frac{3}{2}\right)}}\,\frac{X_c^{\left(\frac{3}{2}\right)}\,(1+X_c)^{\left(\frac{3}{2}\right)}}{(1+2X_c)^{\left(\frac{1}{2}\right)}} \qquad for\ subcritical\ flow \tag{48}$$

where,

$$X_c = \frac{z\,d_c}{B}\ and\ F_{rc} = 1 = \frac{V_c}{\sqrt{g\,d_c}}$$

$$Q_{Tr} = \frac{C_n\,\sqrt{S_o}\,B^{\left(\frac{8}{3}\right)}}{n\,z}\,\frac{X_n^{\left(\frac{5}{3}\right)}\,(1+X_n)^{\left(\frac{5}{3}\right)}}{\left(z+2X_n\sqrt{z^2+1}\right)^{\left(\frac{2}{3}\right)}} \qquad for\ supercritical\ \ flow \tag{49}$$

in which, $X_n = \frac{zd_n}{B}$ and n = Manning roughness coefficient.

C_n = 1.0 in metric units and 1.486 in foot pounds (U.S. customary units). When the theoretical findings of the study were compared to the practical data of other researchers, it was found that the discharge and surface profile results had excellent agreements.

Litsa and Evangelos (1995) considered the flow lines curvatures at the end drop to assess the flow over-passing a drop in channels of trapezoidal cross sections and simulating the flow passing the end drop as flow over-passing a weir of sharp crest. Considering both subcritical and supercritical flows, a generic end depth discharge expression was derived. The discharges that resulted from this expression were evaluated against experimental results from various theoretical methods and surface profiles.

Ferro (1999) derived an important theoretical method for the prediction of (Q_{Tr}) as a function of (d_e) for trapezoidal free drop channels assuming zero pressure at drop and simulating the flow lines at drop as parallel lines neglecting the nappe contraction. The theoretical expression presented by Ferro (1999) was of the form:

$$Q_{Tr} = \left[\,\mu_R\,(\frac{d_e}{B})^{\frac{3}{2}} + \mu_T\,z\,\left(\frac{d_e}{B}\right)^{\frac{5}{2}}\right]B^{5/2}g^{1/2} \tag{50a}$$

where,

$$\mu_R\ =\ F_{rn}[\frac{(2+F_{rn})^{\frac{3}{2}} - {F_{rn}}^{3}}{3F_{rn}}]^{3/2} \tag{50b}$$

and

$$\mu_T = \frac{[-\frac{2{F_{rn}}^3}{3} - \frac{2{F_{rn}}^5}{15} + \frac{2}{15}(2 - {F_{rn}}^2)^{\frac{5}{2}}]^{5/4}}{{F_{rn}}^{1/4}} \quad (50c)$$

in which,

$$F_{rn} = \frac{V_n}{\sqrt{g d_n}} \quad (50d)$$

where, V_n and d_n are the average velocity and uniform depth at uniform flow region, respectively. It should be mentioned that Eq. (50a) is to be used for subcritical flow conditions only. The Froude number at uniform flow region for supercritical conditions of flow should be determined from the expression:

$$F_{rn} = \frac{{S_o}^{1/2}{R_h}^{2/3}}{n\, g^{1/2}{d_n}^{1/2}} \quad (51)$$

where, n = Manning coefficient of roughness and R_h = hydraulic radius at the uniform flow region.

For supercritical flow conditions ($F_{rn} > 1$), the following equation is proposed by Ferro (1999) for the estimation of end depth discharge (Q_{Tr}):

$$Q_{Tr} = A_n \frac{{S_o}^{1/2}{R_h}^{2/3}}{n} \left[\mu_R \left(\frac{d_e}{B}\right)^{\frac{3}{2}} + \mu_T\, z\left(\frac{d_e}{B}\right)^{\frac{5}{2}}\right] B^{5/2}\, g^{1/2} \quad (52)$$

where, A_n = flow cross section at uniform region of flow.

An accurate correlation for the determination of (Q_{Tr}) as a function of (d_e) was presented by Ramamurthy et al. (2004) for free drop in horizontal channels of trapezoidal shape sections considering the impacts of curved flow lines and non-uniform velocity distribution at free fall. The observed static pressure head distribution values showed a good degree of agreement compared with the predicated ones at brink section. Utilizing the static pressure distribution that was recorded, the force due to end section pressure was evaluated.

Beyrami et al. (2006) utilized the principles of momentum and vortex theorems as foundations for the theoretical exploration of free drop in smooth channels having various cross section shapes including channels of trapezoidal cross sections. For smooth trapezoidal channels, curves were produced for the variation of (d_e/d_c) with ($z\, d_e/B$) and ($z\, d_e/B$) with ($Q_{Tr} z^{1.5}/(g^{0.5}B^{1.5}$). They found good agreement between the results of their theoretical method and those of few other previous studies. They found that the range of the coefficients of pressure distribution (K) for trapezoidal channels was between 0.3033 and 0.3558. Therefore, they were unable to provide a particular solution for the variations of (EDD) and (EDR).

Simulation of flow passing over a free drop in channels of trapezoidal shape sections via a (Volume of Fluid) model had been presented by Ramamurthy et al. (2006). Distributions of pressure and velocity as well as the profiles of water surfaces were the main outcomes of the model. The predicted values were verified using previously obtained practical results prevailing fair agreement.

A model based on support vector machines to predict (EDR) and (EDD) correlations for smooth horizontal and three inclined trapezoidal free drop channels was developed by Pal and Goel (2007). Previously obtained empirical relations and a model of neutral network were used to compare the anticipated values of discharge and end depth. The anticipated values of (EDR) and (EDD) showed coefficient of correlations higher than 0.995. Using the machines of support vector very much reduced the time of computations.

Vatankhah (2013) depicted a theoretical approach for the relationship of (EDD) in smooth horizontal trapezoidal free drop channels. Modelling the free flow drop as a flow over-passing a sharp crested weir with crest at bed allowed to obtain two direct discharge equations for subcritical flow. The results obtained from the recommended (EDD) equations showed excellent agreements between predicted flow rates and those observed experimentally.

Noori and Ibrahim (2016) examined impacts of side and bed slopes in smooth trapezoidal channels on the free overfall over a brink section, three different models were fabricated and tested. The beds in the models were 0.1 m wide , 3.7 m long and had four different bed slopes (S_o = 0.0, 0.0033, 0.01 and 0.02) in addition to three different side slopes (z = 0.268, 0.577, and 1). The experimental program's upstream Froude number was in the range of 0.34 to 1.92 for discharges between 0.98 and 16.34 l/s. A linear equation of the following form was found for the variation of ($z\, d_e/ B$) with ($z\, d_c/B$):

$$\frac{z\, d_e}{B} = C_1 \left(\frac{z\, d_c}{B}\right) \tag{53}$$

The value of (d_e/d_c) ratio was represented by the constant C_1 which is mainly dependent on the channel slope and flow Froude number. For smooth horizontal trapezoidal channel beds, C_1 = 0.7911, while, C_1 = 0.729 for Froude numbers less and greater than one in trapezoidal channels having various slopes. Noori and Ibrahim (2016) presented a high determination coefficient linear correlation of the following form for the variations of (d_e/d_c) with (S_o):

$$\frac{d_e}{d_c} = 0.7962 - 8.0369\, S_o \tag{54}$$

Noori and Ibrahim (2016) arrived at an empirical power expression for the change of ($Q_{Tr}\, z^{1.5}/\sqrt{g\, B^5}$) with both ($z\, d_e/B$) and ($S_o$) of the form:

$$\frac{Q_{Tr}\ z^{1.5}}{\sqrt{g\, B^5}} = 6.514\, \left(\frac{z\, d_e}{B}\right)^{1.71} \quad (S_o)^{0.182} \tag{55}$$

Another empirical power equation was also obtained by Noori and Ibrahim (2016) for the change of ($Q_{Tr}\, z^{1.5}/\sqrt{g\, B^5}$) with ($z\, d_e/B$) for horizontal channels (S_o) = 0.0 as:

$$\frac{Q_{Tr}\ z^{1.5}}{\sqrt{g\, B^5}} = 2.239\, \left(\frac{z\, d_e}{B}\right)^{1.702} \tag{56}$$

Ibrahim and Noori (2016) provided a useful experimental analysis to focus on how free overfall in trapezoidal channels was impacted by side slope, bed slope and bed roughness. Three models with 0.1 m wide beds, 3.7 m lengths and side slopes of (z) = 0.268, 0.577, and 1.0 were built and tested. Every model was given four bed slopes (S_o) = 0.0, 0.0033, 0.0067 and 0.02. Strips of dimensions 0.6 cm × 0.6 cm with eight different roughness patterns were used to roughen the bed. The Froude number covered both subcritical and supercritical states of flow and varied from 0.38 to 1.31 during the course of the experimental program. At a roughness concentration of 0.125, Manning's roughness coefficient (n) reached its maximum value at which a linear equation of the following form defined the fluctuations of (d_e/B) with ($z\, d_c/B$):

$$\frac{z\, d_e}{B} = a_1 \left(\frac{z\, d_c}{B}\right) \tag{57}$$

where, a_1= constant gives the value of (d_e/d_c) which was found equal to 0.76 for sloping channels and 0.78 for channels of horizontal beds.

Ibrahim and Noori (2016) depicted a polynomial expression for the variation of (d_e/d_c) with channel bed slope (S_o) of the form:

$$\frac{d_e}{d_c} = C_1 {S_o}^2 + C_2 S_o + C_3 \tag{58}$$

where, C_1, C_2 and C_3 are constants their values depend on the roughness pattern (see Ibrahim, 2012). The combination impacts of ($z\, d_e/B$) and (S_o/n) on ($Q\, z^{1.5}/\sqrt{g\, B^5}$) were taken into account to obtain two helpful empirical power expressions. The first expression was for trapezoidal channels roughened with strips having different bed slopes as:

$$\frac{Q_{Tr}\ z^{1.5}}{\sqrt{g\ B^5}} = 2.573\ \left(\frac{z\ d_e}{B}\right)^{1.711}\ (\frac{S_o}{n})^{0.098} \tag{59}$$

While, the second expression was for horizontal channels having various roughness patterns of bed roughened with strips as:

$$\frac{Q_{Tr}\ z^{1.5}}{\sqrt{g\ B^5}} = 2.173\ \left(\frac{z\ d_e}{B}\right)^{1.684} \tag{60}$$

2.3 Channels of circular sections

Replogle (1962) based his analysis of circular channels on few assumptions applied to the momentum equation of Diskin (1961) developing equivalent momentum equations and demonstrating the residual pressure, factor of energy reduction and factor of momentum correction have little impact on the solution. However, these could explain that about 5% disparity between the discharge predicted by the Diskin's momentum equation and the discharge in reality.

Smith (1962) investigated the free end drop problem in channels of circular cross sections through considering zero pressure and unit momentum coefficient at end drop aiming to get exact theoretical solution. He found that this theoretical solution always gave the predicted area smaller than the real area. Additionally, Smith (1962) provided a dimensionless curve for the relationship between (d_e/d) with ($Q_C/d^{5/2}$) for the estimation of discharge in open-ended free flowing spherical pipes of any size but not exceeding ($Q_C/d^{5/2}$) = 3.7, where, (Q_C) is the end discharge in the circular channel and (d) is the channel diameter.

Diskin (1963) presented a simple power equation for the variation of $Q_C/(gd^5)^{1/2}$ with (d_e/d) of the form:

$$\frac{Q_C}{(gd^5)^{\frac{1}{2}}} = 1.82\ (\frac{d_e}{d})^{1.96} \tag{61}$$

The results of Eq. (61) differ from the results of momentum equation by 0.5% for the range of (d_e/d) between 0.05 and 0.75.

Rajaratnam and Muralidhar (1964) developed an equation for discharges in terms of end drop depth as the result of an analysis of circular channel using the momentum technique as :

$$\frac{Q_C}{(gd^5)^{\frac{1}{2}}} = 1.54\ \left(\frac{de}{d}\right)^{1.84} \tag{62}$$

In case of sloping channels, (d_e/d_c) was found to be dependent on (S_o/S_c) and the value of (d_e/d_c) was decreased from 0.75 to 0.487 for the increase of (S_o/S_c) value between – 4.0 and 8.0.

In order to determine the free end drop depth for circular channels of horizontal beds, Subramanya and Kumar (1993) developed an equation considering the energy and continuity of the water surface profile at end drop. For the horizontal smooth pipes with subcritical flow conditions, they suggested the following expression:

$$6F\left(\frac{d_c}{d}\right) - 4\eta - 3f\left(\eta, \frac{d_c}{d}\right) = 0 \tag{63a}$$

in which,

$$F\left(\frac{d_c}{d}\right) = 1 + 0.0625\ [(2\theta_C - Sin\theta_C)/(Sin\theta_C\ *\ \left(\frac{d_c}{d}\right))] \tag{63b}$$

and

$$f\left(\eta,\frac{d_C}{d}\right) = [0.125(2\theta_C - Sin2\theta_c)^3]/[\left(\frac{d_c}{d}\right)(2\theta_e - Sin2\theta_e)^2 Sin\theta_c] \quad (63c)$$

In equations (63b and 63c), θ_C and θ_e are angles formed by (d_c) and (d_e) related to the cross sectional center of the circle. The study outcome results demonstrated that the end depth ratio (d_e/d_c) was almost constant and equal to 0.73. For the variation of $(Q_C/(gd^5)^{\frac{1}{2}})$ with (d_e/d) a calibration curve was presented using the experimental data available in the literature for the free end drop in smooth and rough horizontal channels of circular cross sections.

Tiwari (1994) produced a computer software and momentum based formulation of free drop in channels of circular cross sections. The calculations took into consideration the weight of the control volume's impact on the slope of the channel. Through the use of the above software, Tiwari (1994) was able to show that the end depth for a circular pipe of diameter (d) with central angle of 20 radians depended on (d_c/d). End depth ratio was found dependent on relative slope (S_c/S_o).

Dey (1998) determined the (EDR) for a smooth circular channel using momentum technique and theoretical analysis for both subcritical and supercritical states of flow. For critical depth diameter ratios up to 0.82 of subcritical state of flow, (EDR) was 0.75. Using Manning's formula, end depth for supercritical flow was expressed as a function of channel slope. The study suggested a discharge equation for subcritical and supercritical states of flow. Additionally, Dey (1998) anticipated the upstream flow profile and studied the free end drop in circular channels of horizontal beds using auto recursive search technique.

A simplified method for the determination of (EDR) for free end drop in channels of circular cross sections having horizontal and mild slopes was proposed by Dey (2001). The flow at the free drop was simulated as a flow over-passing sharp weir with crest height on bed to determine the (EDR) for a circular channel while utilizing the velocity coefficient as a free parameter. Available experimental results were compared well with the theoretical model results. In case of critical depth-diameter ratio up to 0.86, the (EDR) varied practically linearly between 0.72 and 0.74. Dey (2001) proposed an equation for the end depth discharge from his theoretical model.

Sterling and Knight (2001) tested the free end drop in horizontal and sloping circular channels in order to obtain simple procedure for estimating flow rates in terms of relative end depth (d_e/d) and relative critical depth (d_c/d). They tested five channel slopes $(S_o) = 0.0$, 0.001 , 0.004 ,0.009 and 0.016 for the range of (S_o/S_c) between 0.0 and 3.2. Form the experimental results, Sterling and Knight (2001) obtained a straight line correlation between end and critical depths for each channel slope finding that the value of EDR (d_e/d_c) was reduced from 0.7518 to 0.6131 for channel bed slopes from 0.0 to 0.016, respectively for circular channels without a horizontal bed. The end depth ratio was found mainly influenced by relative slope (S_o/S_c) for both subcritical and supercritical flow conditions.

Dey (2003) conducted a laboratory investigation on the free end drop of inverted semicircular channels. He obtained an equation for the end depth ratio using the momentum technique which proved that pressure coefficient was unnecessary. (EDR) = 0.705 for subcritical flow was obtained for the condition that critical depth-diameter ratios not exceeding 0.42. For supercritical flow, however, the end drop depth was presented in terms of (S_o) using Manning's formula. Flow rate was determined for the states of subcritical and supercritical and related to the end drop depth. Dey (2003) performed tests in three inverted semicircular channels having different diameters. The experimental results and projected values agreed well with the exception of supercritical flow conditions where there was slight deviation.

Ahmad (2005) derived a theoretical expression for the determination of flow rate as a function of end depth for the free end drop in inverted semicircular channels considering the flow at the end of the channel as a flow over-passing a weir of sharp crest with zero crest height and considering zero pressure and neglecting the curvature of flow lines at channel brink. He found that the end depth ratio (d_e/d_c) nearly constant equal to 0.713 for the range of (d_c/d) between zero and 0.42 and he presented the following expression for the determination of end depth flow rate for subcritical flow conditions:

$$\frac{Q_C}{g^{1/2}d^{5/2}} = \frac{[sin^{-1}(2.8\left(\frac{d_e}{d}\right)+2.8\left(\frac{d_e}{d}\right)\sqrt{1-8\left(\frac{d_e}{d}\right)^2}]^{3/2}}{8[1-8\left(\frac{d_e}{d}\right)^2]^{1/4}} \tag{64}$$

Eq. (64) was compared with the available experimental data getting $\pm 5\%$ inaccuracy in the predicted results. Additionally, Ahmad (2005) presented a theoretically based set of curves for the prediction of flow rate for supercritical flow conditions in semicircular channels in terms of brink depth, channel bed slope, Manning roughness coefficient and channel diameter.

Nabavi et at. (2009) developed a theoretically based model to determine the flow rate in channels having horizontal and mild sloping beds of inverted semicircular cross sections. The obtained model was analyzed depending on the momentum method in order to obtain a relationship for (EDR) from which the (EDR) was obtained equal to 0.7 for critical-depth ratio not exceeding 0.4. The theoretical model results yielded a useful equation for the determination of end depth discharge. The obtained results of discharge were compared with the available results of previous investigators showing well agreements.

To model free end drop in circular channels, Sharifi et al. (2011) applied the genetic programming (GP). By utilizing a model selection technique and GP on practical results of channels of circular cross sections having flat beds, they were able to get an equation for determining end depth ratio as follows:

$$\frac{d_c}{d_e} = Ae^{B\sqrt{S_o}} \tag{65}$$

Eq. (65) is accurate in dimensions and A and B are coefficients.

Ahmad and Azamathulla (2012) developed a theoretical approach for the perdition of flow rate as a function end drop depth in channels of circular cross section through simulating the channel at brink as zero-height sharp crested weir. They proposed an exponential equation for the estimation of dimensionless flow rate $(Q_C/(g\ d^5)^{\frac{1}{2}})$ as a function of (d_e/d) for subcritical flow conditions of the form:

$$\frac{Q_C}{g^{1/2}d^{5/2}} = 1.866\ (\frac{d_e}{d})^{1.995} \tag{66}$$

Eq. (66) was recommended to be valid for the range of (d_e/d) between 0.01 and 0.85 . The results of this study for subcritical flow conditions were compared with the available experimental data showing excellent agreement. Moreover, Ahmad and Azamathulla (2012) presented a set of curves for the direct solution of predicting dimensionless flow rate as a function of end drop depth, bed slope of the channel and Manning roughness coefficient for supercritical conditions flow. They compared the results of supercritical flow conditions with the available practical results getting $\pm 12\%$ deviations.

In an experimental and theoretical investigation by Rashwan and Idress (2013), the brink depth efficiency as a flow rate monitoring tool in horizontal and mild circular open channels was evaluated. Utilizing the findings, a mathematical expression was obtained using the equations of momentum, flow rate and Froude number. Experimental data were used to calibrate the obtained model. Circular channels may be utilized to precisely determine the discharge rates since the flow rate was calculated from end drop depth and the findings of the lab experiments well matched the calculated values.

Zeidan et al. (2021) made the use of brink depth as useful tool for the measurement of flow rates in horizontal inverted semicircular smooth channels. From the tests of three different channel diameters, the practical laboratory results showed that the (EDR) was varying between 0.7013 and 0.9589 with an average = 0.8201 for the critical depth-diameter ratio not exceeding 0.40. Zeidan et al. (2021) presented two empirical approaches for the estimation of dimensionless discharge. In the direct approach, they proposed the following direct expression for the prediction of discharge:

$$\frac{Q_C}{g^{1/2}d^{5/2}} = 1.2393(\frac{d_e}{d})^{1.4898} \tag{67}$$

Eq. (67) was obtained with a coefficient of determination = 0.9821, while, in the indirect approach, Zeidan et al. (2021) proposed another expression for the estimation of discharge of the form:

$$\frac{Q_C}{g^{1/2}d^{5/2}} = 1.0335\left(\frac{d_e}{d}\right)^{3.0138} \tag{68}$$

Eq. (68) was obtained with a coefficient of determination = 1.0.

The dimensionless critical depth (d_c/d) was correlated to the dimensionless end depth (d_e/d) as in the following equation :

$$\frac{d_c}{d} = 1.1597\left(\frac{d_e}{d}\right) + 0.001 \tag{69}$$

Eq. (69) was obtained with a coefficient of determination = 0.9768. In the above study, it was found that the pipe diameter had no impact on the relationship between end discharge and brink depth.

2.4 Channels of Triangular Sections

Replogle (1962) conducted a study on triangular channels with free overfalls using few assumptions taken by Diskin (1961) in order to produce momentum equations similar to those of Diskin (1961) concluding that correction factors of energy and momentum and the residual pressure did not affect the outcome result more than 5% inaccuracy compared with Diskin's momentum equation results. Moreover, Replogle (1962) presented the measurements of velocity and pressure distribution in the tested channels.

Rajaratnam and Muralidhar (1964) utilized a theoretical momentum-based method in order to test free end drop in exponential channel shapes including triangular free end drop channels. They found that (EDR) = 0.795 for smooth horizontal triangular channels having side slopes (z) = 1.0. Also, Rajaratnam and Muralidhar (1964) concluded that in case of smooth triangular channels, the (EDR) value is dependent on the relative slope of the channel (S_o/S_c).

Tiwari (1994) developed a computer program software to solve the momentum approach expression for free end drop in channels of triangular cross section shapes having horizontal beds. In the derivation, he took into account the weight of the control volume's impact on bed slope, while, taking the channel side slopes as (z = 1). He presented his expression as in the following form:

$$K_1(d_e/d_c)^5 - \left(\frac{5}{2}\right)(d_e/d_c)^2 + \left(\frac{3}{2}\right) = 0 \tag{70}$$

For horizontal smooth bed and neglecting end pressure, (EDR) = 0.7746 and the same value was obtained by Diskin (1961).

Ferro (1999) employed the same assumptions used for the theoretical approach of free end drop problem in rectangular and trapezoidal channels (mentioned before) into the theoretical solution of free end drop of triangular channels. He derived an expression for prediction of flow rate (Q_T) in triangular channels in terms of (d_e) and channel side slopes (z) of the following form for subcritical flow conditions as:

$$Q_T = \mu_T\ g^{1/2}\ z\ {d_e}^{5/2} \tag{71}$$

where, μ_T is defined in Eq. (50c).

For subcritical flow conditions, substitution of F_{rn} = 1 into Eq. (50c) in order to obtain the value of μ_T then substituting μ_T into Eq. (71) yields:

$$Q_T = 1.3595\ g^{1/2}\ z\ {d_e}^{5/2} \tag{72}$$

Additionally, Ferro (1999) derived a theoretical equation for the evaluation of (d_e/d_n) in terms of Froude number (Frn) as:

$$\frac{d_e}{d_n} = \frac{{F_{rn}}^{1/2}}{\left[-\frac{2\,F_{rn}{}^3}{3} - \frac{2F_{rn}{}^5}{15} + \frac{2(F_{rn}{}^3+2)^{5/2}}{15}\right]^{1/2}} \tag{73}$$

(d_e/d_n) = 0.8844 is the result of substituting (F_{rn}= 1) into Eq. (73) for subcritical conditions of flow in smooth triangular free end drop channels. Utilizing the practical results due to Peruginelli (1980) and Ferreri and Ferro (1990), Ferro (1999) validated his theoretical results showing very good agreements.

Beyrami et al. (2006) presented a model to forecast (d_e/d_c) values and to predict (Q_T) for free end drop in channels of triangular shape sections having smooth beds by applying momentum equation and free vortex theorem. The analysis of smooth triangular channels for subcritical flow conditions produced (d_e/d_c) = 0.8051 using the theoretical approach that was described. The equation shown below was offered as a way to estimate (Q_T) as a function of (d_e):

$$Q_T = 1.2548\ g^{\frac{1}{2}}\, z\ d_e{}^{\frac{5}{2}} \tag{74}$$

According to Beirami et al. (2006), the results of their proposed model agreed well compared with the available results of several previous researchers.

Nabavi (2008) proposed a model to anticipate (EDR) in triangular free end drop channels based on the momentum equation. He anticipated the pressure distribution at the channel free end drop in equilateral triangular channel and finding that the (EDR) value for equilateral smooth triangular channels was 0.695 in case the critical depth-channel height ratio did not exceed 0.6 and the flow condition is subcritical. Nabavi (2008) recommended to use Manning equation for the determination (EDR) in terms of (F_{rn}) and (S_o/S_c). He predicted that discharge rate is depending on end depth for subcritical and supercritical flow conditions in triangular channels.

Vatankhah (2015) simulated free end drop in smooth open channels having smooth beds and different cross section shapes (including channels of triangular shapes) via a theoretical approach for calculating (EDR) and (EDD). In this approach, the flow over free end drop was considered a flow over-passing a weir with zero crest height in subcritical states of flow. The equations representing the different channel shapes for determination of (EDR) and (EDD) are shown in Eq. (31) and (32), respectively. Using (a) = 2 in Eq. (31) for triangular channels yields (EDR) = 0.817 and using (a) = 2 and (k) = z (side slopes) in Eq. (32) yields an equation for the determination of (Q_T) in terms of (d_e) and (z) of the form :

$$Q_T = 1.171\ g^{1/2}\, z\ d_e{}^{5/2} \tag{75}$$

Irzooki and Hasan (2018) conducted laboratory tests on smooth and rough triangular free end drop channels, in order to investigate hydraulically the behavior of the free end drop. They conducted their experimental testing program in a flume having 0.3 m width, 0.4 m depth and 6 m length. During the experimental study, different smooth and rough triangular channels were tested having three different side slopes (z), while, four different channel bed slopes (S_o) were tested. In rough channels, three gravel sizes (1.18, 2.36 and 4.75 mm) were tested. (EDR) = 0.788 for smooth triangular channels, while, for rough channel (EDR) values were between 0.7048 and 0.7704 for the values of (F_{rn}) between 0.31 and 1.47. Irzooki and Hasan (2018) concluded that the combined impacts of bed roughness and channel bed slope can be ignored and they presented an equation for (Q_T) in terms (d_e) and (z) of the form:

$$Q_T = 1.252\ g^{1/2}\, z\ d_e{}^{5/2} \tag{76}$$

Abrari et al. (2019) derived a theoretically based model utilizing the geometric center end depth velocity equation of general trapezoidal section for which the energy and continuity equations were combined. Through this theoretical model, the end depth ratio was determined for channels having different shape sections of free end drop for subcritical flow conditions. The fruitful outcome of the proposed model was an expression for the determination of (Q_T) as a function of (d_e) of the following form :

$$\mathrm{Q_T} = 1.201\ \mathrm{g}^{1/2}\ \mathrm{z}\ \mathrm{d_e}{}^{5/2} \tag{77}$$

The value of (EDR) for unconfined smooth and horizontal triangular free end drop channel obtained by Abrari et al. (2019) was 0.809 for subcritical flow conditions. The outcome results of the proposed model were compared with the results of previous researchers prevailing quite good agreements.

Muhsin and Noori (2021) conducted an intensive laboratory study on the characteristics of free end drop in triangular channels of smooth beds via experimenting different models having various channel side slopes and different main slopes of the channel concluding that the correlations for (EDR) were straight lines relationships for all bed slopes and a wide range of (F_{rn}) between 0.64 and 4.15. They determined the value of (EDR) = 0.804 for unconfined horizontal and smooth triangular free end drop channels of side slopes (z) = 1.0. Muhsin and Noori (2021) made an essential conclusion that the main bed slope had a great influence on both (EDR) and (EDD). They presented the following expression for the variation of (d_e/d_c) with the main bed slope (S_o) as:

$$\frac{d_e}{d_c} = 254.94\, {S_o}^2 + 9.4406 S_o + 0.7974 \tag{78}$$

Additionally, Muhsin and Noori (2021) proposed a relationship for the estimation of discharge (Q_T) in terms of (d_e) and (S_o) of the form:

$$Q_T = \left[\frac{0.707}{\left(254.94\, {S_o}^2 + 9.4406 S_o + 0.7974\right)^{\frac{5}{2}}}\right] z\, g^{1/2}\, {d_e}^{5/2} \tag{79}$$

Eq. (79) was extended for predicting (Q_T) in horizontal and smooth triangular free end drop channels to the following form:

$$Q_T = 1.245\, z\, g^{1/2}\, {d_e}^{5/2} \tag{80}$$

All results obtained by Muhsin and Noori (2021) were verified using the theoretical and experimental results of previous researchers prevailing well agreements.

Muhsin (2021) carried out an experimental study on the flow measurement in rough triangular free end drop channels by experimenting twelve different physical models. The models were roughened with strips of 1cm x 1cm cross sections fixed on the model sides in order to have seven patterns of strips of concentrations 0.5, 0.25, 0.125, 0.0625, 0.03125, 0.015625 and 0.0078125. During the experimental program, three different side slopes and four various channel bed slopes were experimented. Muhsin (2021) observed that the correlations between (d_e) and (d_c) were straight lines and values of (d_e/d_c) increased with the decrease of concentration reaching a maximum value at concentration 0.125. A quadratic equation was presented for the variation of (d_e/d_c) with (S_o/n) of the form:

$$\frac{d_e}{d_c} = 0.075\, (S_o/n)^2 + 0.1567((S_o/n\,) + 0.8894 \tag{81}$$

Moreover, Muhsin (2021) obtained an empirical expression for the determination of (Q_T) in terms of (d_e), (S_o) and (n) of the following form:

$$Q_T = \left[\frac{0.707}{\left(0.075\, (S_o/n)^2 + 0.1567\left(\frac{S_o}{n}\right) + 0.8894\right)^{\frac{5}{2}}}\right] z\, g^{1/2} {d_e}^{5/2} \tag{82}$$

Substituting (S_o) = 0.0 into Eq. (82) for horizontal channels yields an expression for the determination of (Q_T) in rough triangular free end drop channels with horizontal beds of the form:

$$Q_T = 0.9477\, z\, g^{1/2}\, {d_e}^{5/2} \tag{83}$$

3. Summary and conclusions

The extensive review of literature undertaken in this paper shows that many of the previous conducted studies were dealing with the free end drop and utilizing it as a very simple and useful key for the determination of discharge rates in channels of different cross sections such as rectangular, trapezoidal, circular and triangular. As a conclusion of this literature review, one may realize that so many works have been undertaken on the properties and behavior of free end drop in rough and smooth rectangular, trapezoidal and circular cross section channels but not very much researches have been conducted and published by hydraulic researchers on the hydraulic and behavior of free end drop in confined or unconfined smooth or rough channels of triangular section shapes for obtaining expressions for the determination of (EDD) in terms of end drop water depth.

Also, this review of literature showed that many expressions had been proposed for the measurement of flow rate and end depth ratio for horizontal and sloping smooth and rough rectangular channels. In these expressions, the main parameters impacting the (EDD) were end drop water depth, channel width and bed slope for channels with smooth beds, while, for rough channels bed roughness was also involved.

Additionally, this paper showed that for smooth trapezoidal free end drop channels, the main factors influencing the end depth discharge were brink depth, channel bed width, channel side slopes and the channel main slope, while, for rough trapezoidal channels, the channel roughness was added to the influencing factors.

Moreover, in case of smooth circular free end drop channels, in addition to the end drop water depth, channel diameter and main slope of the channel were the main parameters impacting the determination of end depth discharge and the impact of channel roughness was undertaken in case of rough circular channels.

Finally, in case of smooth triangular free end drop channels, the present study showed that the parameters affecting the end depth flow rate were end drop water depth, side slopes of the channel and channel main slope, while, for rough triangular channels, the impact of roughness should be added to the affecting parameters.

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