

# Mathematical Modeling for Prediction of Heating and Air-Conditioning Energies of Multistory Buildings in Duhok City

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## ABSTRACT

Present endeavor is devoted to estimate the air-conditioning and heating energies or loads of modern buildings in Duhok City, Iraq using new mathematical models. Many parameters have been considered in current modeling, namely, area of building, number of storeys and types of the common materials of the building walls. Regression analysis is performed to formulate new mathematical linear and nonlinear models for the loads. In addition, Fuzzy logic is utilized in the third model employing Sugeno's regulation. The outcomes reveal that the reasonable matching is achieved between the proposed models and mechanical engineering analytical solutions of heating and air-conditioning standards. Consequently, high correlation coefficient as more than 85% is determined between the predicted values of the models and analytical results. The linear model shows perfect matching with the analytical outputs more than the other proposed mathematical formulations.

**KEYWORDS :** heating and air-conditioning energies, multistory building, mathematical modeling, fuzzy logic, regression analysis.

## 1. INTRODUCTION

The quickened urbanization in human living styles has increased the demand for electrical energy or power utilized in buildings (Chaowen and Dong, 2015). Air-conditioning system in building is accounted for 40%-60% of the total electrical power consumption (Scotton, 2012; Wu, 2012). This energy consumption is affected by the materials used in building construction based on their heat transfer (Committees et al., 2009). The estimation of the required energy for air-conditioning and heating systems in developing countries such as Iraq is essential strategy due to the limitation of energy cost and global warning phenomenon (Housing, 2012; Joudi, 1996). Nowadays, tedious methods are used to predict the required energy for air-conditioning and heating loads in buildings. Accordingly, many parameters must be taken into account in these methodologies such as the type of constructional materials and their coefficients of heat transfer, outdoor

and indoor temperatures and humidity, number of occupants (persons), lightening, etc (Committees et al., 2009; Housing, 2012). Many works have been recently launched to estimate the required electrical power for air-conditioning and heating systems of buildings in various zones over the world. Mathematical models have been introduced in those studies using different statistical and mathematical concepts such as multiple linear regression, nonlinear regression, fuzzy logic analysis, etc. Several investigations (Al-Shallawi, 2004; Catalina et al., 2013; Catalina et al., 2008; Chou and Bui, 2014; Dong et al., 2005; Hui, 1997; Jain et al., 2014; Korolija et al., 2013; Lam et al., 2010; Li et al., 2009a, b; Puchkal and Jurmanov, 2013; Wu and Sun, 2012; Yiu and Wang, 2007) have been published concerning the mathematical modeling of heating and air-conditioning energies of buildings via statistical regression analysis. Fuzzy logic mathematical model is also utilized to predict these loads by other researchers (Costa and La Neve, 2015; Dexter and Benouarets, 1996; Fraise and Virgone, 1997; Huang et al., 2009; Kolokotsa et al., 2006; Li et al., 2011; Lopez et al., 2004; Ngo and Dexter, 1999; Rezeka et al., 2015; Xu and Zhou, 2012).

The formulation of simple mathematical models to evaluate air-conditioning and heating loads of buildings is necessary to be used in lieu of aforementioned ponderous methods. The proposed models should be compatible with the modern commonly used materials

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in construction and latitude of the building location. There are still no remarkable easiest mathematical models proposed for consumed loading for air-conditioning and heating of buildings in Duhok City, Iraq. Thus, further investigations in this direction are essential to include the effect of the commonly used building materials and recent outdoor temperatures in this region. The main objectives of the current work involve the formulation of new simple mathematical models in order to compute the thermal energy required for air-conditioning and heating systems of buildings in Duhok City, Iraq. Different shapes of common multistory buildings are considered, namely, frame and bearing wall buildings, in addition to various local masonry materials are such as heavy solid concrete block, hollow concrete block, siporex and clay brick. Statistical and computational approaches in SPSS and MATLAB softwares are utilized as multiple linear regression, nonlinear regression and fuzzy logic. Linear regression functions, nonlinear regression functions and fuzzy model are proposed to facilitate the solution of air-conditioning and heating loads with realizing the required accuracy.

## 1. METHODOLOGY

The methodology of present investigation is composed of two parts, data collection of heating and air-conditioning loads, which are required for modeling and analyzing the data to formulate new mathematical models for prediction the loads.

### 1.1 Calculation of Heating and Air-Conditioning Loads

Heating and cooling thermal energies required for air-conditioning and heating systems of many multistory buildings in Duhok City are computed using mechanical engineering specification (Kreider et al., 2009). The diversity in the height and area of the building as well as type of common constructional masonry materials were considered. The design of the study depends on the recent extremes of the local high and low outdoor temperatures at Duhok City. Three plans of buildings (Figs. 1-3) are utilized in the current study. The properties of the selected buildings with these plans are given in Table 1. Two forms of multistory buildings are considered in the analysis, frame building and bearing wall building. In Duhok City, the modern buildings are featured by average maximum height of 10, 4 and 2 storeys for frame buildings, bearing wall buildings and ordinary residential buildings, respectively. This range of height was applied in present mathematical modeling of heating and cooling system load of buildings. The direction of the buildings with respect to the north is illustrated in Figs. 1-3. It is expected that the required energy for cooling these buildings is more than for heating due to high outdoor temperature record in

Duhok City with latitude of 36.90N.

Nowadays, different building materials (see Table 2) are widely used in Duhok City for walling construction to decrease the consumption of air-conditioning and heating in buildings. The effect of these wall materials is considered on present energy consumption modeling.

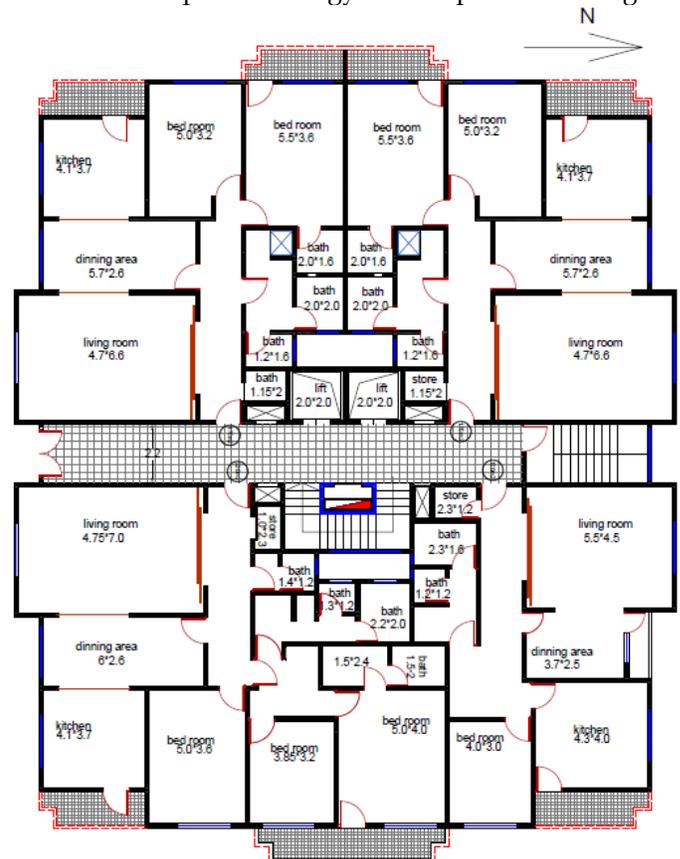


Fig (1) : Plan for buildings with height range of one to ten storeys

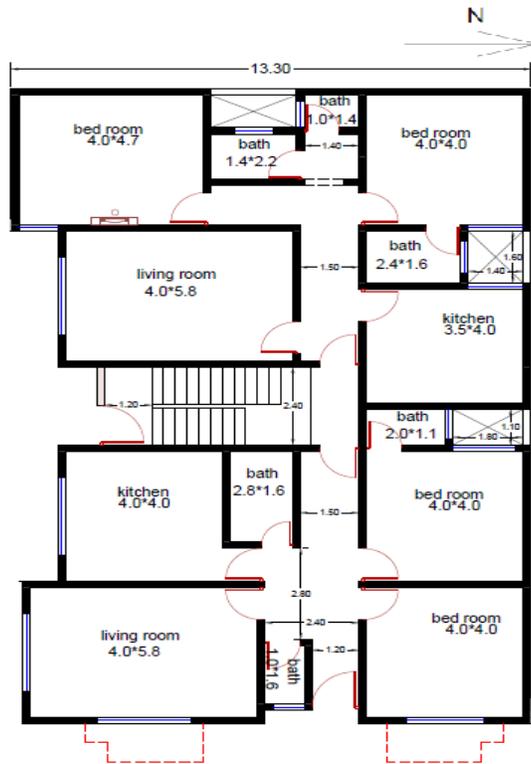


Fig (2) : Plan for buildings with height range of one to four storeys

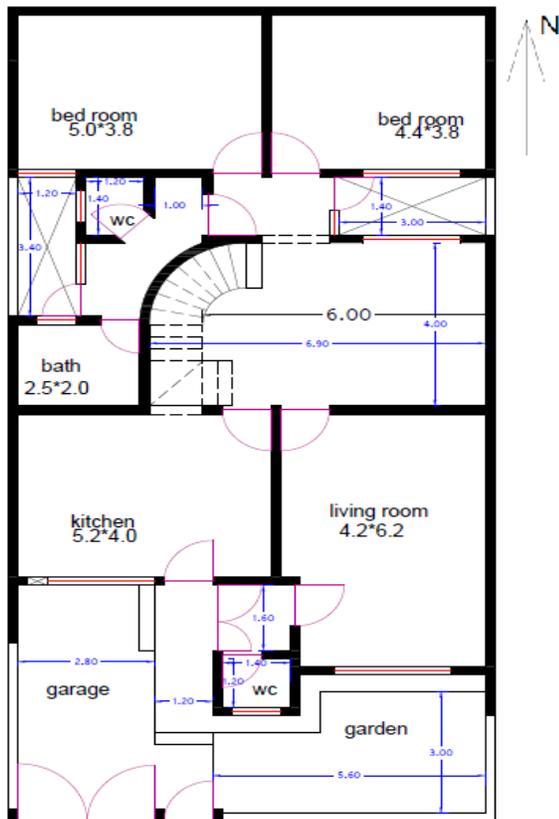


Fig (3) : Plan for buildings with height range of one to two storeys

TABLE (1) : Characteristics of present buildings

Building No.	Area of construction (m <sup>2</sup> )	Height of storey (m)	Number of storeys
1	642.4	3	1
2	642.4	3	2
3	642.4	3	3
4	642.4	3	4
5	642.4	3	5
6	642.4	3	6
7	642.4	3	7
8	642.4	3	8
9	642.4	3	9
10	642.4	3	10
11	220	3	1
12	220	3	2
13	220	3	3
14	220	3	4
15	150	3	1
16	150	3	2

TABLE (2) : DENSITY OF THE COMMONLY USED MATERIALS IN MASONRY

Masonry material	Density (kN/m <sup>3</sup> )
Concrete solid block	20.8
Concrete hollow block	16
Siporex	9
Clay brick	14

In current thermal energy determination, a proper set of conditions is introduced such as solar influences, outdoor and indoor temperatures, building function and humidity. The calculation of heating load is performed with consideration of critical design temperature corresponding with slight or no heating from solar radiation. Consequently, the outdoor temperatures per each hour (see Table 3) of the recent coldest day (29/Jan/2016) were obtained and used in the current findings. The critical design temperatures of cooling were chosen from the recent warmest day (20/July/2016) in Duhok City. The hourly temperature recording of these days is tabulated in Table 3. Thus, the lowest and highest temperatures of -5.65 and 46.2°C, respectively have been used in present air-conditioning loading calculations. The relative humidity is another effective factor on the air-conditioning and heating loads design. Duhok City is characterized by low and relatively high humidity in summer and winter, respectively. The outdoor humidity obtained in present analysis was 63.3% for the coldest day and 18.5% for warmest day according to recent meteorological record

in Duhok City. Indoor temperature and humidity are adopted according to comfortable requirements for human (Committees et al., 2009; Housing, 2012). Accordingly, the comfortable operative temperature and relative humidity were chosen as 24°C and 50%, respectively. The required thermal energy (CL) for cooling system of building is calculated as (Kreider et al., 2009) :

$$CL = CL_{WA} + CL_C + CL_P + CL_{inf} + CL_L + CL_e + CL_{WI} + CL_f \tag{1}$$

where  $CL_{WA}$  is cooling load for walls,  $CL_C$  is cooling load for ceiling,  $CL_P$  is cooling load for persons,  $CL_{inf}$  is cooling load for ventilation and infiltration,  $CL_L$  is cooling load for lightening,  $CL_e$  is cooling load for equipment and furniture,  $CL_{WI}$  is cooling load for windows and  $CL_f$  is cooling load for floor. The heat transmission through walls is highly affected on the air-conditioning load during summer due to the

difference between outdoor and indoor temperatures. This external heat gained through walls is determined using thermal conductivity of these walls. The present buildings were built by walls with section shown in Fig. 4. The unit thermal resistance of the wall (1/U) can be computed as (Jones, 2005):

$$\frac{1}{U} = \frac{1}{h_i} + \frac{1}{h_o} + \sum_{i=1}^n \frac{\Delta x_i}{k_i} \tag{2}$$

where U is the unit conductance (overall heat transform coefficient) in W/m<sup>2</sup>.K or kcal/m<sup>2</sup>.h.oC,  $\Delta x_i$  is thickness of iconstructional layer which form the wall as given in Fig. 4,  $h_i$  and  $h_o$  are the coefficient of heat transfer for air film inside and outside the wall in W/m<sup>2</sup>.K respectively,  $k_i$  is the thermal conductivity coefficient for i material within the wall section and n is the number of constructional layers within the wall.

TABLE (3) : HOURLY TEMPERATURES FOR RECENT COLDEST AND WARMEST DAYS IN DUHOK CITY

Time	Outdoor temperature(°C) for 29/Jan./2016	Outdoor temperature (°C) for 20/July/2016
1:00	-1.65	27.25
2:00	-2.5	27.2
3:00	-3.6	30.1
4:00	-3.65	31
5:00	-5.15	29.85
6:00	-5.65	31.35
7:00	-3.6	31.6
8:00	-5.9	33.45
9:00	-6.4	36.8
10:00	-3.1	40.6
11:00	2.65	43.1
12:00	5.05	44.35
13:00	5.75	45.05
14:00	6.6	46.1
15:00	7.25	46.2
16:00	7.6	45.95
17:00	7.7	45.4
18:00	7.25	45
19:00	5.85	43.95
20:00	3.35	42
21:00	0.25	39.3
22:00	-1.6	37.3
23:00	-2.35	35.25
24:00	-1.95	33.45

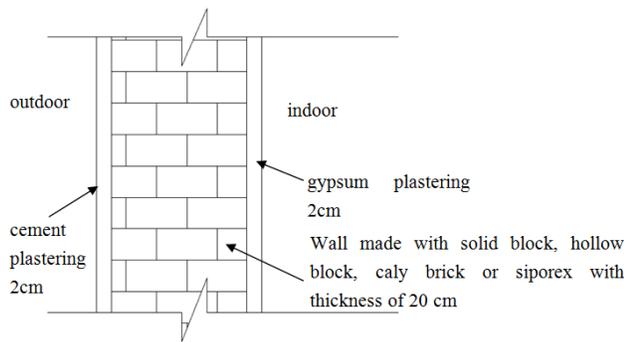


Fig (4) : Section of the widely used wall of the buildings in Duhok City

The aforementioned wall unit thermal resistance is used to find the required cooling loading for wall heat gain as (Committees et al., 2009; Housing, 2012; Jones, 2005; Kreider et al., 2009; Ministry of Planning, 2013; Yousif, 2009) :

$$CL_{WA} = U_{WA} \cdot A_{WA} \cdot \Delta T_{WA} \quad (3)$$

where  $A_{WA}$  is the area of walls exposed to outdoor temperature in  $m^2$ ,  $\Delta T_{WA}$  is the difference between indoor and outdoor temperatures with consideration of the correction factor to compensate the solar temperatures on the buildings as (Housing, 2012):

$$\Delta T = (CLTD + LM) \cdot k_c + (25.5 - T_i) + (T_o - 29.4) \quad (4)$$

where  $LM$  is the correction coefficient for the latitude and month,  $k_c$  is the correction coefficient for wall color,  $CLTD$  is the cooling load temperature difference, and  $T_i$  and  $T_o$  are the design indoor and outdoor temperatures respectively.

$T_i$  temperature is equal to 24oC and  $T_o = T_m - DR/2$  where  $DR$  is the daily range of outdoor temperature within 24 hr.

The heat gain for the ceiling, which is exposed to external temperature has been calculated in the cooling load as (Housing, 2012) :

$$CL_c = U_c \cdot A_c \cdot \Delta T_c + \quad (5)$$

where  $U_c$  is the ceiling unit conductance ( $W/m^2.K$ ) for widely used ceiling section (Fig. 5) in Duhok City,  $A_c$  is the area of roof exposed to outdoor temperature in  $m^2$ ,  $\Delta T_c$  is the difference between indoor and outdoor temperatures with consideration of correction factor for ceiling to compensate the solar temperatures on the buildings as (Housing, 2012) :

$$\Delta T_c = [(CLTD_c + LM_c) \cdot k_{cc} + (25.5 - T_i) + (T_o - 29.4)] \cdot F \quad (6)$$

where  $LM_c$  is the correction coefficient for roof,  $k_{cc}$  is the correction coefficient for roof color = 1.0,  $CLTD_c$  is the cooling load temperature difference and  $F$  is a factor equals 0.75 for roof with fake ceiling or suspended ceiling as in present study.

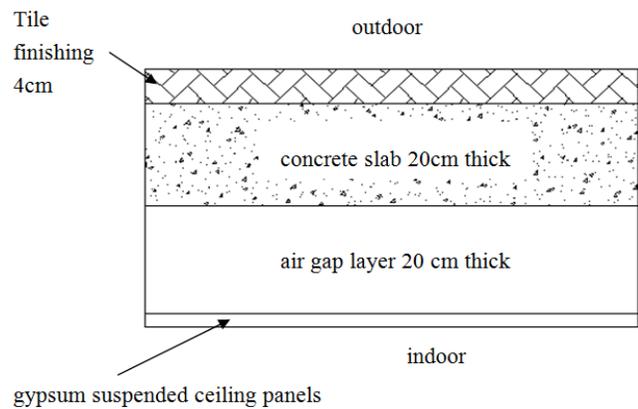


Fig (5) : Section of the commonly used roof of the buildings in Duhok City

The cooling energy due to heat gain through floor is determined in present study as (Housing, 2012):

$$CL_f = U_f \cdot A_f \cdot \Delta T_f \quad (7)$$

where  $U_f$  is the floor unit conductance in  $W/m^2.K$  for widely used floor section (Fig. 6) in Duhok City,  $A_f$  is the area of floor exposed to outdoor temperature in  $m^2$  and  $\Delta T_f$  is the difference between indoor and outdoor temperatures.

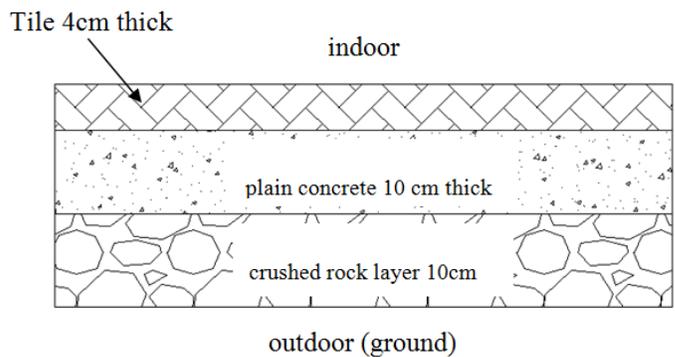


Fig (6) : Section of the widely used floor of the buildings in Duhok City

The external heat gain by the plastic doors can be overcome with air-conditioning using the following expression (Housing, 2012):

$$CL_D = U_D \cdot A_D \cdot \Delta T_D \quad (8)$$

where  $U_D$  is the door unit conductance of 4.66  $W/m^2.K$  for widely used door section in Duhok City =,  $A_D$  is the area of the doors exposed to the outdoor temperature in  $m^2$  and  $\Delta T_D$  is the difference between indoor and outdoor temperatures as used for walls in oC.

Glass windows induce heating gain in two effects, namely, radiation and transmission. These effects are considered in present cooling load calculation as follows (Housing, 2012) :

$$CL_{WI} = CLTD_{WI} \cdot U_{WI} \cdot A_{WI} + A_{WI} \cdot S_c \cdot SHG \cdot CLF \quad (9)$$

and

$$CLTD_{WI} = (CLTD)_T + (25.5 - T_i) + (T_o - 29.4) \quad (10)$$

where  $U_{WI}$  is the windows overall heat transfer coefficient of 4.71 W/m<sup>2</sup>.K,  $A_{WI}$  is the area of glass in m<sup>2</sup>,  $S_c$  is the shading coefficient,  $SHG$  is the solar gain and  $CLF$  is the cooling load factor for time. The cooling load for ventilation and infiltration is represented as a part of internal heat gain and given as (Housing, 2012):

$$CL_{inf} = CL_{infs} + CL_{inf1} \tag{11}$$

and

$$CL_{infs} = 1.22V_{vent} \cdot (T_o - T_i) + 1.22V_{inf} \cdot (T_o - T_i) \tag{12}$$

$$CL_{inf1} = 2940V_{vent} \cdot (W_o - W_i) + 2940V_{inf} \cdot (W_o - W_i) \tag{13}$$

where  $CL_{infs}$  and  $CL_{inf1}$  are the sensible load and latent load by infiltration and ventilation in W respectively,  $V_{vent}$  and  $V_{inf}$  are the average air discharge and  $W_o$  and  $W_i$  are outdoor and indoor relative humidity in percent respectively.

$V_{vent}$  can be calculated as:

$$V_{vent} = V_{vent}/\text{person} \cdot \text{Number of persons} \tag{14}$$

where  $V_{vent}/\text{person}$  is the required amount of air ventilation per person.

Number of persons = 8 per each unit (i.e. the maximum number of persons for most apartments or residential units in Duhok City).

$$V_{inf} = N \cdot V_{volume} / 3600 \tag{15}$$

where  $N$  is the number of air change which is dependent on the function of building = 1.0 in present study. The occupants cooling load (CLP) is regarded as internal heat gain. In present work, this load is calculated as (Housing, 2012) :

$$CL_p = N_p \cdot \frac{q_s}{\text{person}} \cdot CLF_p + N_p \cdot \frac{q_1}{\text{person}} \tag{16}$$

where  $q_s/\text{person}$  is the sensible average heat provided by a person depending on his/her activity and equal to 72 W,  $q_1/\text{person}$  is the latent average heat provided by a person depending on his/her activity and equal to 60 W,  $CLF_p$  is the coefficient of cooling load per person and

$N_p$  is the number of persons and equal to 8.

The cooling load needed to overcome the heating gain of each equipment depends on its efficiency. In present investigation, an approximate magnitude of this load per unit meter square of area is utilized as 6.8 W/m<sup>2</sup> (Committees et al., 2009).

The radiant heat from lightening is introduced in the thermal load of cooling system as (Housing, 2012):

$$CL_L = N_L \cdot W_L \cdot F_b \approx \frac{25W}{m^2} \times \text{floor area} \tag{17}$$

where  $N_L$  is the number of lights,  $W_L$  is the power of light (W/m<sup>2</sup>) and  $F_b$  is a factor of 1.0 and 1.2 for normal lights and fluorescent, respectively.

In present study, CLL has been found using 25 W/m<sup>2</sup> for lightening as given in (17).

The thermal energy (HL) for heating system of buildings can be determined in W as (Housing, 2012):

$$HL = HL_p \cdot HL_L \cdot HL_e - HL_{WA} - HL_{WI} - HL_D - HL_C - HL_f - HL_{inf} \tag{18}$$

where  $HL_p$ ,  $HL_L$ ,  $HL_e$ ,  $HL_{WA}$ ,  $HL_{WI}$ ,  $HL_D$ ,  $HL_C$ ,  $HL_f$  and  $HL_{inf}$  are the heating load in W for persons, lightening, equipment, wall, windows, doors, ceiling, floor, ventilation and infiltration respectively. The heating load used in current work is twice of floor area as recommended by Dong (Dong et al., 2005). In Eq.18,  $\Delta t$  is calculated using indoor temperature of 24oC and lowest outdoor temperature in Duhok City of -6.4oC. The outdoor humidity of 63.3% recorded in 29/1/2016 has been used in heating load calculations without correction. Consequently, no need to use CLTD in  $\Delta t$  because there is no solar heat gain in winter. Thermal energies for cooling and heating systems in W for selected buildings are determined using (1) and (18) (see Table 4 and Table 5). Four types of common constructional materials of walls are considered for these buildings. The utilized procedure in this energy calculation is tedious due to the consideration of many abovementioned factors in the mechanical engineering standards.

TABLE (4) : Cooling load in W for the selected buildings in Duhok City

Buildi ng No.	Solid concrete block wall	Hollow block concrete	Siporex wall	Clay brick wall
1	150135.213	147562.432	137868.782	142509.142
2	271340	266194.394	246807.1	256087.814

3	392544.76	384826.36	355745.406	369666.486
4	513749.522	503458.318	464683.72	483245.158
5	634953.97	622090.28	573622.03	596823.83
6	756158.67	740722.242	682560.342	710402.502
7	877363.37	859354.204	791498.654	823981.174
8	998568.07	977986.166	900436.966	937559.846
9	1119772.77	1096618.128	1009375.278	1051138.518
10	1240977.47	1215250.09	1118313.59	1164717.19
11	67515.1	55646.155	40307.5	43801.72
12	125122.6	121199.91	90522.6	97511.04
13	182730.1	176846.06	130830.1	141312.76
14	240337.6	232492.22	171137.6	185114.48
15	44675.87	42926.49	36470.65	39637.326
16	82596.6	79097.84	66186.16	72519.512

TABLE (5) : Heating load in W for the selected buildings in Duhok City

Building No.	Solid concrete block wall	Hollow block concrete wall	Siporex wall	Clay brick wall
1	106325.244	102452.6	89669.43	97517.87
2	145847.77	138185.67	112536.15	128233.03
3	185370.3	173877.15	135402.87	158948.19
4	224892.83	209568.63	158269.6	189663.35
5	264415.3661	245052.1461	181136.2961	220378.4961
6	303937.8966	280702.0326	204003.0126	251093.6526
7	343460.4272	316351.9192	226869.7292	281808.8092
8	382982.9577	352001.8057	249736.4457	312523.9657

9	422505.4882	387651.6922	272603.1622	343239.1222
10	462028.0188	423301.5788	295469.8788	373954.2788
11	47449.5	44403.3	31302	37475.64
12	72021.36	65928.96	39726.36	52073.64
13	96593.22	87454.62	48150.72	66671.64
14	121165.08	108980.28	56575.08	81269.64
15	35850.61	33254.55	24685.28	29946.63
16	56102.82	50910.7	33772.16	44294.86

**2. MATHEMATICAL MODELING**

The calculated data for cooling and heating loads (Table 4 and Table 5) have been utilized in current models formulation. The simplicity and efficiency are considered in model proposition. Three expressions are adopted for these mathematical models, namely, linear regression, nonlinear regression and fuzzy logic models.

**2.1 Regression Models**

The regression analysis method is usually used to predict a variable  $Y$  as a function of the other parameters  $X_1, X_2, \dots, X_n$  (Catalina et al., 2013; Catalina et al., 2008; Graybill and Iyer, 1994). The regression functions can be categorized into main and sub-forms. The multiple regression (polynomial) models are employed in present work to relate a dependent variable with two or more independent parameters in linear or nonlinear mathematical expression as follows (Al-Abadi, 2011; Chatterjee and Hadi, 2006; Freedman, 2009; Graybill and Iyer, 1994):

$$Y = C_0 + C_1X_1 + C_2X_2 + C_3X_3 \tag{19}$$

$$Y = C_1X_1^{C_1}X_2^{C_2}X_3^{C_3} \tag{20}$$

The main difference between multiple linear and nonlinear regression functions is that the linear regression function is simultaneously linear with respect to unknown constant parameters  $C_i$ , while nonlinear regression function is not.

In present mathematical modeling, the equation of heating load (QH) and cooling load (QC) are formulated depending on the regression analysis of aforementioned energy outputs. In addition, two input parameters were taken into account, namely, floor or constructional area of the building and number of storeys as listed in Table 1. This analysis has been carried out in SPSS statistics 20 software. Accordingly,

linear multiple regression function is formulated to find heating and cooling loads as:

$$Q = C_0 + C_1X_1 + C_2X_2 + C_3X_1X_2 \tag{21}$$

where  $Q$  is the heating or cooling load in W,  $X_1$  is the area of floor in  $m^2$  and  $X_2$  is the number of storeys.

Four types of building materials are considered in wall construction of these buildings such as solid concrete blocks, hollow concrete blocks, siporex and clay brick, which are widely used in Duhok City. The constant parameters ( $C_0, C_1, C_2$  and  $C_3$ ) in the multiple regression equations are determined in SPSS software as follows (Chatterjee and Hadi, 2006):

$$\begin{bmatrix} Q_1 \\ Q_2 \\ \vdots \\ Q_n \end{bmatrix} = \begin{bmatrix} 1 & A_{f1} & N_1 & A_{f1} & N_1 \\ 1 & A_{f2} & N_2 & A_{f2} & N_2 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 1 & A_{fn} & N_n & A_{fn} & N_n \end{bmatrix} \begin{bmatrix} C_0 \\ C_1 \\ C_2 \\ C_3 \end{bmatrix} + \varepsilon \tag{22}$$

or

$$[Y] = [X][C] + \varepsilon \tag{23}$$

where  $\varepsilon$  is the error.

The principle of least sum of square error (Chatterjee and Hadi, 2006; Graybill and Iyer, 1994) has been used to obtain the constants as:

$$\sum_{i=1}^n \varepsilon_i^2 = \sum_{i=1}^n (y_i - C_0 - C_1A_{fi} - C_2N_i - C_3A_{fi}N_i) \approx zero \tag{24}$$

Then,

$$[X][C] = [Y] \rightarrow [X]^T[X][C] = [X]^T[Y] \tag{25}$$

Thus, four equations in the form of Eq. 21 have been formulated with the multiple regressions for heating or cooling load prediction as:

1. Cooling load in W:

a. for concrete solid block walls building:

$$Q_c = -13510.654 + 67.259A_f + 27361.91N + 145.929A_fN \tag{26}$$

b. for concrete hollow block walls building:

$$Q_C = -20204.913 + 77.134A_f + 27936.199N + 141.103A_fN \tag{27}$$

c. for siporex walls building :

$$Q_C = -5703.647 + 53.28A_f + 6568.428N + 159.439A_fN \tag{28}$$

d. for clay bricks walls building :

$$Q_C = -5906.886 + 53.614A_f + 9508.993N + 162.083A_fN \tag{29}$$

2. Heating load in W :

a. for concrete solid block walls building :

$$Q_H = -2720.597 + 108.464A_f + 17364.584N + 34.462A_fN \tag{30}$$

b. for concrete hollow block walls building :

$$Q_H = -2291.644 + 107.902A_f + 14651.67N + 32.646A_fN \tag{31}$$

c. for siporex walls building :

$$Q_H = 4513.46 + 96.565A_f - 59.144N + 35.739A_fN \tag{32}$$

d. for clay bricks walls building :

$$Q_H = 3575.97 + 98.107A_f + 5441.906N + 39.382A_fN \tag{33}$$

Nonlinear regression models for heating and cooling loads are also formulated using expression (34) with parameters  $C_4$  and  $C_5$ .

$$Y = C_4(X_1X_2)^{C_5} \tag{34}$$

The unknown parameters in the proposed nonlinear equations were determined by a popular least squares method too. Thus, the summation of total errors between predicted values of load, calculated by proposed equations and found by analytical method is minimized. Accordingly, eight equations (i.e. Eqs. (35-42)) are formulated using nonlinear regression analysis in SPSS statics 20 software for the given data in Table 4 as follows :

1. Cooling load in W :

a. for concrete solid block walls building:

$$Q_C = 419.021(A_fN)^{0.91} \tag{35}$$

b. for concrete hollow block walls building:

$$Q_C = 398.785(A_fN)^{0.913} \tag{36}$$

c. for siporex walls building:

$$Q_C = 281.938(A_fN)^{0.944} \tag{37}$$

d. for clay bricks walls building:

$$Q_C = 304.748(A_fN)^{0.94} \tag{38}$$

2. Heating load in W :

a. for concrete solid block walls building:

$$Q_H = 991.707(A_fN)^{0.697} \tag{39}$$

b. for concrete hollow block walls building:

$$Q_H = 981.585(A_fN)^{0.689} \tag{40}$$

c. for siporex walls building:

$$Q_H = 787.153(A_fN)^{0.675} \tag{41}$$

d. for clay bricks walls building:

$$Q_H = 810.438(A_fN)^{0.697} \tag{42}$$

The statistical analysis has been carried out in SPSS software to find the significance of the coefficients in proposed mathematical models. In these models, 95% confidence interval (upper and lower bounds) of each coefficient was determined. Consequently, it is demonstrated that all parameters are located within this interval, which confirms their significance in the proposed equations. The accuracy of the aforementioned proposed equations for cooling and heating system of buildings is checked via the difference between model predicted outputs and analytical outcomes (Table 4 and Table 5). Thus, the validity of these models has been demonstrated in terms of coefficient of determination ( $R^2$ ) and root mean square deviation (RM) as (Korolija et al., 2013):

$$RSS = \sum_{i=1}^n (y_i - \hat{y}_i)^2 \tag{43}$$

$$TSS = \sum_{i=1}^n (y_i - \bar{y})^2 \tag{44}$$

$$R^2 = 1 - \frac{RSS}{TSS} \tag{45}$$

$$RM = \sqrt{MSE} = \sqrt{\sum_{i=1}^n \frac{(y_i - \hat{y}_i)^2}{n}} \tag{46}$$

where RSS = residual sum of error squares, TSS is the total sum of error squares which is proportional to observed data variance,  $y_i$  is the observed data (loading in W),  $\bar{y}$  is the mean of the whole observed data set (i.e.  $\bar{y} = \frac{\sum_{i=1}^n y_i}{n}$ ),  $\hat{y}$  is the predicted data by proposed models (loading in W),  $n$  is the number of samples in the data set ( $n = 16$  in the present work) and MSE is the mean square error. The adjusted coefficient of determination ( $R_{aj}^2$ ) is computed as well to show the strength of the proposed mathematical models with respect to the used parameters within each model as (Souza et al., 2013) :

$$R_{aj}^2 = R^2 - \left(\frac{P - 1}{n - P}\right) (1 - R^2) \tag{47}$$

where,  $P$  is the number of parameters or coefficients in the proposed model.

The coefficient of determination and adjusted coefficient of determination for the proposed heating and cooling loads are determined as listed in Table 6. Reasonable matching is observed between the predicted load values and those found by analytical method according to high coefficients of determination that are approximately equivalent to one.

TABLE (6) : Coefficients of determination for the proposed linear and nonlinear equations

Eq.	RSS	TSS	R <sup>2</sup>	RM	P	n	R <sup>2</sup> <sub>aj</sub>
26	238946929	2.47E+12	1	3864.48	4	16	1
27	77768465.05	2.24E+12	1	2204.66	4	16	1
28	9689310.33	3.04E+11	1	778.19	4	16	1
29	16739836.5	2.06E+11	1	1022.86	4	16	1
30	4492584602	2.47E+12	1	16756.7	4	16	1
31	491418457.9	2.24E+12	1	5541.99	4	16	1
32	1139351391	3.04E+11	1	8438.57	4	16	1
33	1031026901	2.06E+11	1	8027.4	4	16	1
35	167974732.3	2.38E+12	0.998	3240.13	2	16	0.9979
36	80344379.49	2.08E+12	0.998	2240.88	2	16	0.9979
37	6645884.525	2.56E+11	1	644.49	2	16	1
38	31027568.2	1.32E+11	1	1392.56	2	16	1
39	3812262501	2.38E+12	0.996	15435.88	2	16	0.9957
40	390826153.1	2.08E+12	0.997	4942.33	2	16	0.9968
41	887626057.1	2.56E+11	0.987	7448.26	2	16	0.9861
42	1730728232	1.32E+11	0.995	10400.51	2	16	0.9946

The validity of present proposed regression models is also examined via Akaike Information Criterion (AIC), which includes the application of parsimony principle to choose the best mathematical model. Accordingly, the model with lower AIC value is adopted to be the best parameterized mathematical model. AIC magnitude is calculated as (Akaike, 1974; Al-Taee, 2009; Burnham and Anderson, 2004; Souza et al., 2013) :

$$AIC = n \ln \left( \frac{RSS}{n} \right) + 2P \tag{48}$$

The AIC values for the current proposed mathematical models are given in Table 7. Lower values for AIC are achieved with using linear models.

TABLE (7) : AIC for the proposed linear and nonlinear models

Linear proposed models								
E	26	27	28	29	30	31	32	
A	27	26	25	25	221	21	239	
Nonlinear proposed models								
E	35	36	37	38	39	40	41	
A	31	31	27	28	293	29	300	

**2.2 Fuzzy Logic Models**

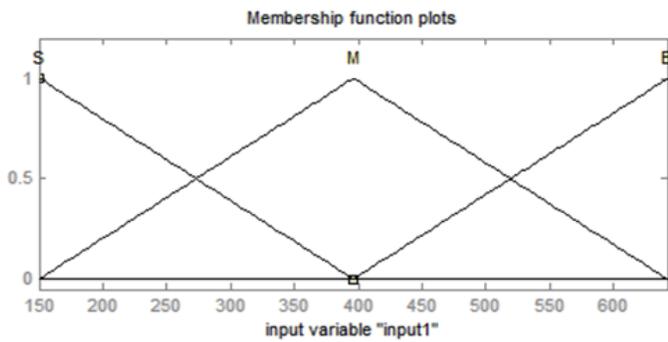
Sugeno fuzzy logic approach is computationally more efficient than the Mamdani fuzzy approach and copes

well with adaptive techniques and suitable for modeling nonlinear systems (Negnevitsky, 2005). In present mathematical modeling of heating and cooling energies in W, the Sugeno fuzzy logic approach has been used with constant output membership functions. Rules were employed to connect the inputs and output of fuzzy model using weighted output level. Consequently, the final output of the logic system is the weighted mean for all rules outputs as given hereunder (Sugeno, 1985) :

$$\text{Final output} = \frac{\sum_{i=1}^k w_i z_i}{\sum_{i=1}^k w_i} \tag{49}$$

where  $z_i$  is the output level,  $w_i$  is firing strength of the rule and  $k$  is the number of rules.

In present fuzzy mathematical models for heating and cooling system loads, two inputs have been used, namely, the area of building floor and number of storeys. Each input is represented by three linguistic variables. These linguistic variables (i.e. Small (S), Medium (M) and Big (B)) with triangle memberships functions (see Figs. 7 and 8) are utilized for the inference mechanisms. Here, triangle membership functions are used because they can offer good representation of the expert knowledge and significantly simplify the process of computation [43].



**Small (S) floor area function is:**

$$-x/246 + 1 \quad \text{for } 150 \text{ m}^2 \leq x \leq 396 \text{ m}^2$$

**Medium (M) floor area function is:**

$$x/246 - 0.61 \quad \text{for } 150 \text{ m}^2 \leq x \leq 396 \text{ m}^2$$

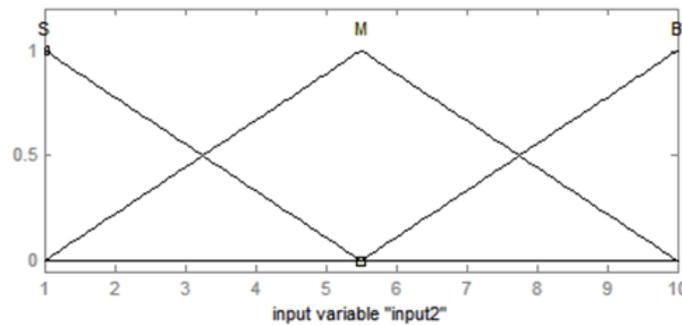
$$-x/246 + 2.61 \quad \text{for } 396 \text{ m}^2 \leq x \leq 642 \text{ m}^2$$

**Big (B) floor area function is:**

$$x/246 - 1.61 \quad \text{for } 396 \text{ m}^2 \leq x < 642 \text{ m}^2$$

Fig (7) : Membership functions for buildings floor area (input 1) for present Sugeno fuzzy model

The output follows zero-order Sugeno mathematical model (constant). Nine constant functions are utilized per each output model with different constant values. Fuzzy logic toolbox in MATLAB has been utilized to predict the loading for cooling and heating system of the selected buildings. The outputs are computed with present fuzzy models in terms of heating and cooling loads in W for the adopted buildings in Duhok City as given in Table 8 and Table 9. The validity of these models is checked via calculation of RM and R2 of the outcomes as listed in Table 10. The difference has been computed between the outcomes of current fuzzy models and analysis outputs for air-conditioning loading (see Table 4 and 5). It has been found that all fuzzy models have considerable coefficient of determination larger than 85%. It is worth to mention that the adjusted coefficient of determination is not applicable to fuzzy models.



**Small (S) number of storeys function is:**

$$-x/4.5 + 1 \quad \text{for } 1 \leq x \leq 5.5$$

**Medium (M) number of storeys function is:**

$$x/4.5 - 0.22 \quad \text{for } 1 \leq x \leq 5.5$$

$$-x/4.5 + 2.22 \quad \text{for } 5.5 \leq x \leq 10$$

**Big (B) number of storeys function is:**

$$x/246 - 1.22 \quad \text{for } 5.5 \leq x \leq 10 \text{ m}^2$$

Fig (8) : Membership functions for number of storeys (input 2) for present Sugeno fuzzy model

TABLE (8) : Cooling load in W for the selected buildings in Duhok City

Building No.	Solid concrete block wall	Hollow block concrete wall	Siporex wall	Clay brick wall
1	151000	147000	132000	137000
2	273000	267000	242000	253000
3	396000	387000	353000	368000
4	518000	507000	463000	483000

5	641000	627000	574000	598000
6	761000	744000	683000	712000
7	879000	859000	790000	824000
8	996000	974000	898000	936000
9	1110000	1090000	1010000	1050000
10	1230000	1200000	1110000	1160000
11	58500	55100	46600	49600
12	111000	108000	87700	93400
13	164000	161000	129000	137000
14	217000	213000	170000	181000
15	42900	39600	32400	34900
16	84100	80200	61800	66700

TABLE (9) : Heating load in W for the selected buildings in Duhok City

Building No.	Solid concrete block wall	Hollow block concrete wall	Siporex wall	Clay brick wall
1	98100	93500	75300	85300
2	141000	132000	102000	119000
3	183000	171000	128000	153000
4	226000	210000	154000	187000
5	268000	249000	180000	221000
6	308000	285000	205000	253000
7	344000	318000	227000	282000
8	381000	351000	249000	311000
9	418000	384000	271000	340000
10	454000	418000	294000	370000
11	43800	41300	31300	36600
12	66800	64000	42200	52300
13	89800	82700	53200	67900
14	113000	103000	64100	83600
15	34300	32200	23700	28100
16	53900	49700	31900	40600

TABLE (10) : Coefficient of determination and root mean square error for present fuzzy models

Used Fuzzy Models	$R^2$	RM
Model for cooling load of buildings with solid concrete block walls	0.90550626	117075.6982
Model for cooling load of buildings with hollow concrete block walls	0.90659641	114325.8453
Model for cooling load of buildings with siporex block walls	0.9127167	103392.3083

Model for cooling load of buildings with clay bricks walls	0.91175573	107926.7575
Model for heating load of buildings with solid concrete block walls	0.861108	49805.7176
Model for heating load of buildings with hollow concrete block walls	0.859466	45959.5658
Model for heating load of buildings with siporex block walls	0.858462	33178.8938
Model for heating load of buildings with clay bricks walls	0.863499	40701.6926

### 3.3 Comparison among the Proposed Mathematical Models

The outputs for aforementioned linear, nonlinear and fuzzy mathematical models are compared to analytical outcomes of cooling and heating loads of the selected multistory buildings. The comparison has been performed with employing histograms in SPSS statics 20 software. The abscissa in this diagram is referring to the ratio of model output and analytical outcome; while the ordinate is representing the frequency of the x-value. The model with good x-mean, which is closest to unity and lower standard deviation, is regarded as the best choice. In present study, sixteen buildings are considered per each walling type. Accordingly, 128 loading values are determined with linear, nonlinear and fuzzy models with consideration of both heating and cooling loads. Current histograms (Figs. 9-11) indicate that the linear model with the mean of 1.0 and standard deviation (std. dev.) of 0.037 is the best model. In general, the proposed models can be considered reasonable in the estimation of loading for heating and cooling loads of the modern multistory buildings in Duhok City.

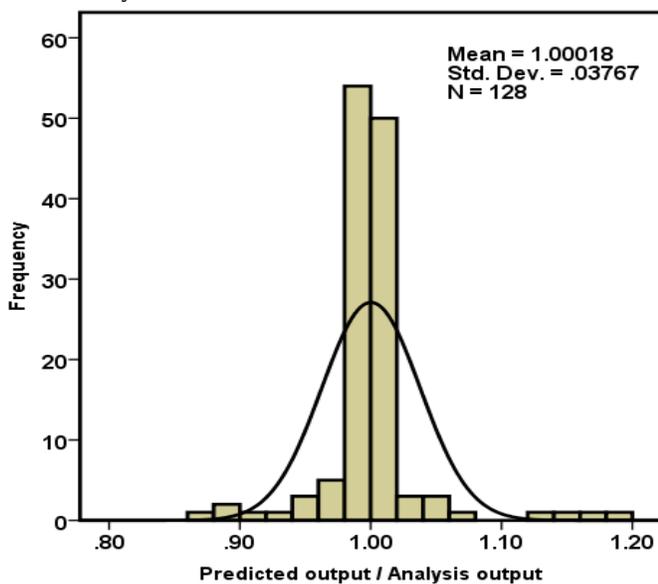


Fig (9) : Histogram for the ratio of predicted load output from linear models and analytical outcome of the load

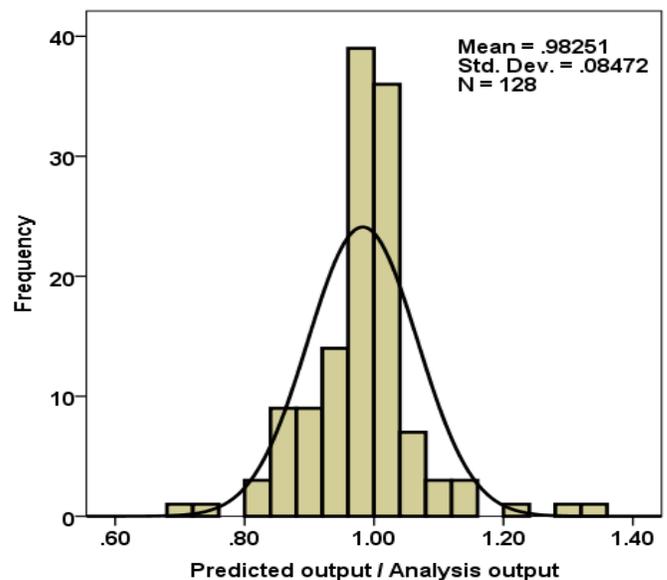


Fig (10) : Histogram for the ratio of predicted load output from nonlinear models and analytical outcome of the load

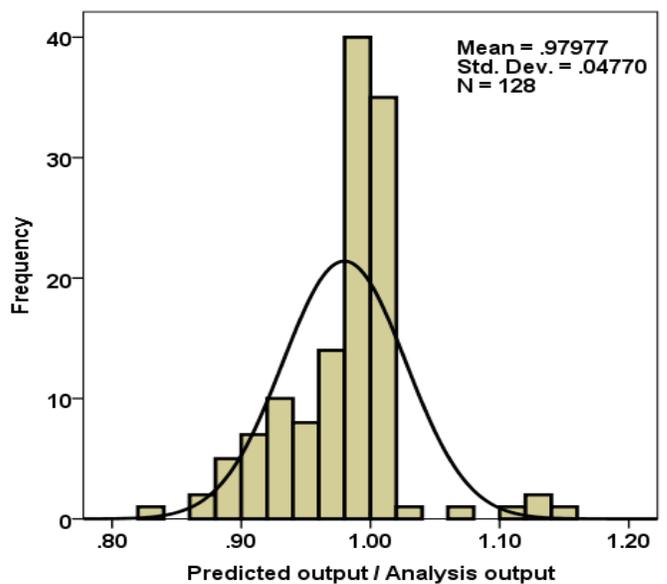


Fig (11) : Histogram for the ratio of predicted load output from fuzzy models and analytical outcome of the load

#### 4. CONCLUSIONS

Three mathematical models were formulated in present investigation, namely, multiple regression linear, regression nonlinear and fuzzy models to predict the heating and cooling loads of common buildings in Duhok City, north of Iraq. Four types of constructional materials used in walls, area of building and number of storeys were considered in current mathematical formulations. According to this mathematical modeling, it is concluded that all the mathematical models to predict the heating and cooling loads of the buildings in Duhok City were valid. The results showed also that linear equations provide higher correlation than the other models with the exact analytical solution outcomes. In addition, best quality of statistical models was achieved via the proposed linear equations with respect to AIC criteria.

#### REFERENCES

- Akaike, H., (1974), A new look at the statistical model identification. *IEEE Transactions on Automatic Control* 19, 716-723.
- Al-Abadi, M.M.Y., (2011), Using information of population in estimating parameters of multiple regression models based on quantile regression with application. *Iraqi Journal for Statistical Sciences* 19, 233-248.
- Al-Shallawi, S.K.H., (2004), Using Mathematical Models for System of Air-conditioning of Building in Mosul City. Mosul.
- Al-Tae, F.A., (2009). Smoothing and prediction for time series using transformation with application, Smoothing and prediction for time series using transformation with application, Second Scientific Conference of Mathematics, Statistics and Informatics, Mosul, Iraq.
- Burnham, K., Anderson, D., (2004), Multimodel Inference: Understanding AIC and BIC in Model Selection. *Sociological Methods & Research* 33, 261-304.
- Catalina, T., Iordache, V., Caracaleanu, B., 2013. Multiple regression model for fast prediction of the heating energy demand. *Energy and Buildings* 57, 302-312.
- Catalina, T., Virgone, J., Blanco, E., (2008), Development and validation of regression models to predict monthly heating demand for residential buildings. *Energy and Buildings* 40, 1825-1832.
- Chaowen, H., Dong, W., (2015), Prediction on Hourly Cooling Load of Buildings Based on Neural Networks. *International Journal of Smart Home* 9.
- Chatterjee, S., Hadi, A.S., (2006), *Regression Analysis by Example*, Fourth ed.
- Chou, J.-S., Bui, D.-K., (2014), Modeling heating and cooling loads by artificial intelligence for energy-efficient building design. *Energy and Buildings* 82, 437-446.
- Committees, A.T., Groups, T., Groups, T.R., (2009), *ASHRAE Handbook-Fundamentals* (SI). American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc.
- Costa, H.R.d.N., La Neve, A., (2015), Study on application of a neuro-fuzzy models in air conditioning systems. *Soft Computing* 19, 929-937.
- Dexter, A.L., Benouarets, M., (1996), A generic approach to identifying faults in HVAC plants. American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., Atlanta, GA (United States).
- Dong, B., Cao, C., Lee, S.E., (2005), Applying support vector machines to predict building energy consumption in tropical region. *Energy and Buildings* 37, 545-553.
- Fraisse, G., Virgone, J., (1997), Thermal comfort of discontinuously occupied building using a classical and a fuzzy logic approach. *Energy and Buildings* 26, 303-316.
- Freedman, D.A., (2009), *Statistical Models: Theory and Practice*. Cambridge University Press, New York.
- Graybill, F.A., Iyer, H.K., (1994), *Regression analysis: concepts and applications*. Duxbury Press, Belmont, CA.
- Housing, M.o.C., (2012), Iraq Blog for Air-conditioning, Iraq.
- Huang, G., Wang, S., Xu, X., (2009), A robust model predictive control strategy for improving the control performance of air-conditioning systems. *Energy Convers Manage* 50, 2650-2658.
- Hui, S.C.M., (1997), A randomised approach to multiple regression analysis of building energy simulation, BS 1997 - 5th Int. IBPSA Conference, Prague, Czech Republic.
- Jain, R.K., Smith, K.M., Culligan, P.J., Taylor, J.E., (2014), Forecasting energy consumption of multi-family residential buildings using support vector regression: Investigating the impact of temporal and spatial monitoring granularity on performance accuracy. *Applied Energy* 123, 168-178.
- Jones, W.P., (2005), *Air Conditioning Engineering*. Elsevier Butterworth-Heinemann.
- Joudi, K.A., (1996), *Principles of air-conditioning and refrigeration*, Second ed. University of Basrah, Basrah, Iraq.
- Kolokotsa, D., Saridakis, G., Pouliezios, A., Stavrakakis, G.S., (2006), Design and installation of an advanced EIB™ fuzzy indoor comfort controller using Matlab™. *Energy and Buildings* 38, 1084-1092.
- Korolija, I., Zhang, Y., Marjanovic-Halburd, L., Hanby, V.I., (2013), Regression models for predicting UK office building energy consumption from heating and cooling demands. *Energy and Buildings* 59, 214-227.

26. Kreider, J.F., Curtiss, P.S., A., R., (2009), Heating and Cooling of Buildings: Design for Efficiency, Second ed. CRC Press.
27. Lam, J.C., Wan, K.K.W., Liu, D., Tsang, C.L., (2010), Multiple regression models for energy use in air-conditioned office buildings in different climates. *Energy Convers Manage* 51, 2692-2697.
28. Li, K., Su, H., Chu, J., (2011), Forecasting building energy consumption using neural networks and hybrid neuro-fuzzy system: A comparative study. *Energy and Buildings* 43, 2893-2899.
29. Li, Q., Meng, Q., Cai, J., Yoshino, H., Mochida, A., (2009a), Applying support vector machine to predict hourly cooling load in the building. *Applied Energy* 86, 2249-2256.
30. Li, Q., Meng, Q., Cai, J., Yoshino, H., Mochida, A., (2009b), Predicting hourly cooling load in the building: A comparison of support vector machine and different artificial neural networks. *Energy Convers Manage* 50, 90-96.
31. Lopez, A., Sanchez, L., Doctor, F., Hagra, H., Callaghan, V., (2004), An evolutionary algorithm for the off-line data driven generation of fuzzy controllers for intelligent buildings, 2004 IEEE International Conference on Systems, Man and Cybernetics (IEEE Cat. No.04CH37583), pp. 42-47 vol.41.
32. Ministry of Planning, R.o.I., (2013), Technical Specifications for Civil Work, Iraq.
33. Negnevitsky, M., (2005), *Artificial Intelligence A Guide to Intelligent Systems*, 2nd ed. Addison-Wesley.
34. Ngo, D., Dexter, A.L., (1999), A robust model-based approach to diagnosing faults in air-handling units. American Society of Heating, Refrigerating and Air-Conditioning Engineers, Inc., Atlanta, GA (US); Univ. of Oxford (GB).
35. Puchkal, V., Jurmanov, B., (2013), Stochastic Model of the Thermal Regime and Heat Consumption of Residential Buildings for Heating. *World Applied Sciences Journal* 23, 191-196.
36. Rezek, S.F., Attia, A.-H., Saleh, A.M., (2015), Management of air-conditioning systems in residential buildings by using fuzzy logic. *Alexandria Engineering Journal* 54, 91-98.
37. Scotton, F., (2012), Modeling and Identification for HVAC Systems, KTH Electrical Engineering, Stockholm, Sweden.
38. Souza, L.d.A., Carneiro, P.L.S., Malhado, C.H.M., Silva, F.F.e., Silveira, F.G.d., (2013), Traditional and alternative nonlinear models for estimating the growth of Morada Nova sheep. *Revista Brasileira de Zootecnia* 42, 651-655.
39. Sugeno, M., (1985), *Industrial applications of fuzzy control*, First ed. Elsevier Science Ltd.
40. Wu, S., Sun, J.-Q., (2012), A physics-based linear parametric model of room temperature in office buildings. *Building and Environment* 50, 1-9.
41. Wu, X.T., (2012), Considerations in energy efficient design of HVAC systems. *Journal of HV&AC* 7.
42. Xu, J., Zhou, J.j., 2012. The intelligent control of the central air conditioning, 2012 International Conference on Computer Science and Information Processing (CSIP), pp. 691-694.
43. Yiu, J.C.-M., Wang, S., (2007), Multiple ARMAX modeling scheme for forecasting air conditioning system performance. *Energy Convers Manage* 48, 2276-2285.
44. Yousif, K.Y., (2009), Production of new lightweight concrete with studying of its mechanical and thermal properties. *Iraqi Journal of Civil Engineering* 1.