

# Contact Stress Analysis of Spur Gear Under the Different Rotational Speed by Theoretical and Finite Element Method

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## ABSTRACT

In this study that spur gears are chosen, contact stress of spur gear is presented under the effect of rotational speed. Three-dimensional simulation of dynamic analysis of gears designed and modeled using ANSYS software. The dynamic analysis included in the determination of dynamic stresses analysis. Contact stress is theoretically calculated and analyzed and numerically estimated using both Hertzian mathematical model and finite element method respectively. Different values of rotational speed used to study its effect on contact stress. Both methods compared by evaluating the percentage error of contact stress, and the modeling of the spur gear and stress analysis of spur gear carried out using SOLID WORK and ANSYS V14, respectively. The most significant note in this study concludes that increasing speed causes vibration and pitting failure due to repetitions.

**KEYWORDS :** Spur gear, Contact stress, Finite Element Method, ANSYS, Rotational speed, Hertzian model.

## 1. INTRODUCTION

Gear is a cylindrical wheel tooth cut that meshed with another gear for transmitting power from one shaft to another, of which both rotate to opposite directions during motion. Gear can also be defined as a rotational toothed wheel that is attached to a shaft. Gears are typically classified to spur, helical, bevel, double helical, hypoid, crown, pinion and rack, epicyclical, worm gears, etc. In the modern engineering systems, they are practically applied in tremendous industries; from a tiny wristwatches to huge machinery equipment, such as; rolling, automobile, aerospace industry, transmitting machinery, hoisting and marine engines (Narayankar & Mangrulkar, 2017). Spur gear is the simplest type of gear used for transmitting power because of its straight teeth cut on the rim parallel to the axis of rotation. It is the most common and cost-effectiveness type of gear. Spur gear is used in oscillating sprinklers, electric screwdrivers, windup alarm clocks, cloth dryers and washing machines (Sainath). The contact between two teeth follows the line of action. The driving wheel begins at the tooth root and moves towards the top, while the following wheel moving reversely from top to

root, as shown in Fig. 1. When a Contact is placed between the root and the pitch circle of the driving wheel in the first half of the meshing, the friction force moved towards the root of the driving tooth and the rotation counteracts. When the contacted teeth passed through pitch circle, the friction force alters direction and acts to the rolling motion. Also, this direction reverses to the followed teeth. Thus, relative motion is fixed across the contact width but is also changed during the interaction. The schematic diagram of the motion of the spur gear is illustrated in Fig. 1. (Damtie & Tilahun, 2014).

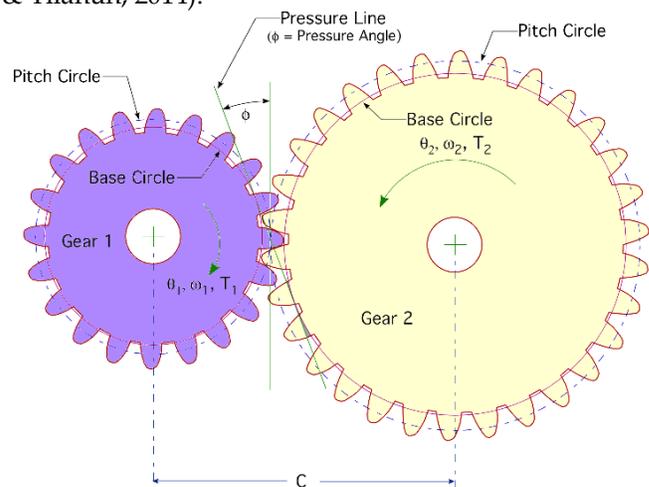


Fig (1) : Schematic diagram of spur gear motion

The main problem in spur gear encountered during rising loads and speeds. By effect of contacting between the two rods at the contact point, it creates a very high

stress that the material cannot withstand it. Reducing the dynamic effect is the main rewards to help remaining the velocity ratio constant which gives rise to increase stress, noise and vibration (Ravindra, 2013). The Normal or resultant force acts along the pressure line at pitch point that solves both tangential and vertical component in a horizontal and vertical plane. The tangential force component is used to calculate torque and power of spur gear. The contact stress and bending stress are induced on the gear due to the tangential load acts on the gear. If contact stress on the gear is higher than the wear strengths of the gear material, gear failure that would take place called as wear or pitting failure of the gear (Chor & Pillai, 2015). When loads are applied to the bodies, the point of contact is, elastically deformation, occurs near their surfaces. The highest stresses happen in regions where the lines collected closest together. The highest stress occurs in two locations, when the force acts in contact point and at the fillet area near the base of a tooth. Surface failure of tooth gear generally occurs due to pitting and scoring. Pitting is actually the fatigue failure of the tooth surface due to many repetitions of the stress of gear surface during motion. The primary property of the gear tooth that provides resistance to pitting is the Hardness. In other words, when a pair of teeth is transmitting a power the fatigue failure takes place on the gear teeth during the repetitions of high contact stress, thus pitting occurs (Rajaprabakaran & Ashokraj, 2013). The other primary cause of gear tooth failure is the existing large tensile stresses in the root fillet of the loaded gear tooth. These stresses decrease the overall gear life and can result in disastrous tooth failure under peak load condition (Abattini, Mirza, & Pawar, 2017; K. Gupta & Chatterjee, 2018). The contact stress is calculated by the method of Hertzian equation based on the contact between two cylinders. In addition, the Finite Element Method is used in order to analyze the contact stress. Hassan (2009) and Hwang et. al., (2013) presented a research study for analyzing contact stress among two spur gear teeth various contact positions of gears during rotation. The study developed a platform for plotting a pair of teeth in contact. Through running the platform for rotating each 3° of pinion from the first position of the contact to the last position of the contact, 10 cases produced. The research represented a sequence position of contact between these two teeth for each case. The graphical results between the position and location of contact during rotation were plotted. Finite element models were made for these cases and stress analysis was done using ANSYS. The both of finite element analysis and Hertzian equation were used for calculating the contact stresses. The finite element method results compared with theoretical calculations (Hassan, 2009; Hwang, Lee, Lee, Han, & Lee, 2013).

Gupta and Verde explained that the study of contact stress is an important parameter for gear design. Analytical methods for calculating gear contact stresses is studied using Hertz's equations and ANSYS, and the results were compared with Hertzian theory. In their paper, they calculated contact stress for various values of modules and concluded that the module is significant geometrical parameter during the design of gear and observed that contact stress decreases with increasing module. The contact stresses are higher at the pitch point (B. Gupta, Choubey, & Varde, 2012). Karaveer et. al., (2013) studied the stress analysis of mating teeth of spur gear to calculate maximum contact stress in the gear teeth. The gained results from Finite Element Analysis (FEA) were compared with theoretical values of Hertzian equation. The spur gears used in this research are made from steel and Grey cast iron. Spur gear stress modeled by ANSYS and calculated by both Finite Element Method (FEM) and theoretically using Hertzian equation. The deformations of Grey cast iron and steel were gained as the efficiency of the gear depends on their deformation. The research results showed that the difference between maximum contact stresses calculated from Hertz equation and Finite Element Analysis is very close and is acceptable (Karaveer, Mogrekar, & Joseph, 2013). Rao et. al., (2015) realized that the key parameter of gear design can be considered where it is contacted stress and deformation. They noticed that when the working stress exceeds the maximum stress, the gears become a failure. They showed that the complex design problem of spur gears for analyzing and modeling require a fine software skill. They compared both methods, concluding that the range of contact stresses and deformation are useful in the selection of material in different applications, which leads to accept the FEM results (P. Rao, Sriraj, & Farook, 2015). Rao and Vamsi (2016) presented analysis and design of the contact stresses of spur gear in a different value of software tools like pro-e and ANSYS. The ANSYS design for sketching the spur gear is used and analyzed the contact stress in mechanical ANSYS multi-physics (P. S. Rao & Vamsi, 2016). Some other researchers implemented a finite element model for stress analysis of gear drives based on multi-point constraints see (Gonzalez-Perez & Fuentes-Aznar, 2017). A finite element model of a gear pair was presented dynamic transmission error (DTE), which was suited to DTE analysis in consideration of gear eccentricities and varying load see (Gonzalez-Perez & Fuentes-Aznar, 2017). Where some researchers investigated, experimentally and analytically, the influence of pitch deviations on the loading capacity of spur gears (Franulovic, Markovic, Vrcan, & Soban, 2017). On the other hand, the external spur gear root bending stress studied using ANSYS finite element analysis and strain

gauge techniques (Lisle, Shaw, & Frazer, 2017). Recently an optimization variables consisted of the gear module, the face width, the pinion and wheel profile shift coefficients and the number of teeth of the pinion showed by (Miler, Žeželj, Lončar, & Vučković, 2018). Then many researchers were studied the contact stress of spur gear using finite element method, ANSYS, and theoretical Hertzian equations; some of them were presented the effect of parameters such as changing pressure angle, teeth number, hardening, and/or materials that cause the failure like fatigue or pitting and minimizing the stresses nevertheless; speed ratio is another parameter for reducing the contact stress which causes fatigue failure and reduces the life of gear. Therefore, the aim of this study is to present the effect of rotational speed of gear on contact stress and comparison between the finite element numerical solution by ANSYS and theoretical Hertzian method. In the next section, the theoretical background and mathematical equations of the Hertzian method explained and then the results and discussion showed, while the conclusion of this research presented in the last section.

**2. THEORETICAL BACKGROUND**

On transfer of power between gears takes place at the contact between the acting teeth, the stresses at the contact point can be computed by means of the Hertzian equation. According to the above mentioned theory, the stresses and deformations of curved bodies in contact can be expressed mathematically. A model applied to the gear-two parallel cylinders in contact is shown in Fig. 2 (Quadri & Dolas, 2015).

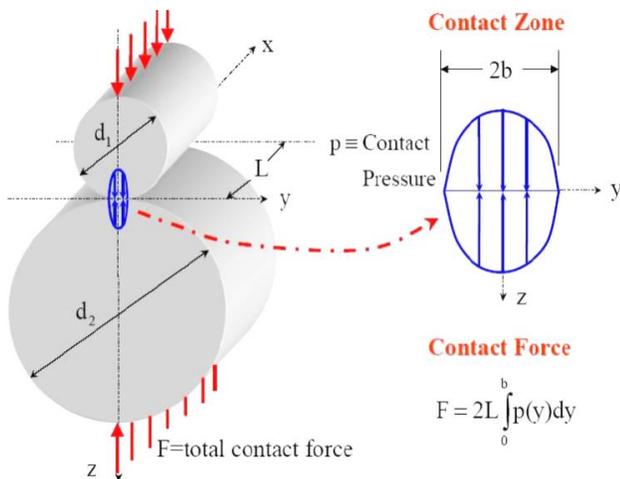


Fig (2) : Hertzian Theory (Quadri & Dolas, 2015)

Based on the Hertzian theory, the simplest expression of the mathematical equation can be explained as follows, the half-width *b* is given by the equation :

$$b = \sqrt{\frac{2F (1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2}{\pi L (1/d_1 + 1/d_2)}}$$

Assuming that the contacting elements are two cylinders of length *L* and cylinder diameters *d*<sub>1</sub> and *d*<sub>2</sub>, as shown in Fig. 2, width 2*b* and length *L* is face width represents the dimensions of the narrow rectangular area of contact, *F* is contact force, *v*<sub>1</sub>, *v*<sub>2</sub>, *E*<sub>1</sub>, *E*<sub>2</sub> Poisson’s ratio and Modulus of elasticity of two materials respectively (Budynas & Nisbett, 2008; Khan, Mangla, & Din, 2015), and the distribution of pressure is elliptical, the maximum pressure can be calculated by :

$$\text{Pressure} = \frac{2F}{\pi b L}$$

The maximum value of contact stress in gear teeth according to the Hertzian equation can be estimated by :

$$\sigma_c = \sqrt{\frac{F}{\pi L} \frac{[1/r_1 + 1/r_2]}{[(1-\nu_1^2)/E_1 + (1-\nu_2^2)/E_2]}}$$

Where, *L* Face width of pinion in mm, *r*<sub>1</sub>, *r*<sub>2</sub> represents the radius of the two gears in mm.

**3. DETERMINATION OF CONTACT STRESS BY HERTZIAN EQUATION**

The contact stress equation for a pair of teeth contacting in contact point, the *r*<sub>1</sub>, *r*<sub>2</sub> should be replaced by the radius of curvature at the pitch point (Khan et al., 2015). Therefore

$$r_1 = \frac{d_{pp} \sin \phi}{2} \text{ and } r_2 = \frac{d_{pg} \sin \phi}{2}$$

Where  $\phi$  represents the pressure angle, *d*<sub>pp</sub>, *d*<sub>pg</sub> represents the pitch circle diameter of pinion and gear respectively. A pair of gears have the same geometry so:

$$d_{pp} = d_{pg} = d_p$$

*d*<sub>p</sub> represent the Pitch circle diameter in mm.

$$\text{so, } r_1 = r_2 = r = \frac{d_p \sin \phi}{2} = r_p \sin \phi$$

$$\text{hence, } \sigma_c^2 = \frac{FE}{\pi L r_p (1-\nu) \sin \phi}$$

$$F = \frac{F_t}{\cos \phi}$$

$$\sigma_c^2 = \frac{E}{\pi (1-\nu)} \left[ \frac{F_t}{L r_p \sin \phi \cos \phi} \right]$$

Same material is used for both pinion and gear such as structural steel.

$$\text{so, } E_1 = E_2 = 200 \text{ GPa, and } \nu_1 = \nu_2 = 0.3$$

$$\sigma_c = 301,572 \left[ \frac{F_t}{Lr_p \sin \phi \cos \phi} \right]^{1/2}$$

The contact stress can be evaluated mathematically by equation 7 for structural steel for a pair of gear teeth (gear and pinion have same geometry and dimensions). The applied torque on the gear can be determined by:

$$T = F_t \times \frac{d_p}{2}$$

The tangential component of the resultant force can be determined as:

$$F_t = \frac{2T}{d_p}$$

$F_t$  is the tangential component of the resultant force. The maximum allowable contact stress after considering the safety factor between the pinion and the gear can be expressed by:

$$\sigma_{ca} = \frac{\sigma_c}{S.F.}$$

Where  $S.F.$  is safety factor and  $\sigma_{ca}$  is the maximum allowable stress.

and the power transmitted  $P$  in  $kW$  between the two gears can be estimated by the following equation:

$$P = T \times \omega$$

Where  $\omega$  is the angular velocity and then:

$$P = \frac{2\pi n_p T}{60 \times 10^6}$$

Where  $T$  represents the transmitted torque and  $n_p$  represents the rotational speed of the pinion (Jabbour & Asmar, 2015; Karaveer et al., 2013).

The mathematical calculation was made based on the following dimensions of the gear and the pinion, as explained in Table 1.

TABLE (1) : Spur gear and pinion dimensions

Part dimension	Symbol	value
No.of teeth	Z	20
Pressure angle	$\Phi$	20
Module	M	6.35
Pitch circle diameter	$d_p$	127
Face width	L	25.4
Dedendum circle diameter	$d_a$	139.7
Addendum circle diameter	$d_d$	111.76

All dimensions are in mm

Now, calculating the torque, tangential component of the resultant force and contact stress between the spur gears (pinion and gear) based on the dimensions in Table 1 can be determined by equation 7, 8, and 10

assuming that the power transmitted is 10 KW, if the rotational speed is 3500 rpm.

$$T = 27283.7 \text{ Nmm} \tag{7}$$

$$F_t = 429.665 \text{ N}$$

$$\sigma_c = 274.0134 \text{ MPa}$$

Noting that the safety factor is 2.8288 which is the same value that taken in ANSYS data.

Then the maximum allowable stress can be determined according to the equation 9.

$$\sigma_{ca} = 96.9 \text{ MPa}$$

#### 4.DETERMINATION OF CONTACT STRESS IN ANSYS:

##### 4.1. GEAR MODELING

The 3D gear geometry is designed with the dimensions shown in Table 1 using ANSYS as shown in Fig.3.

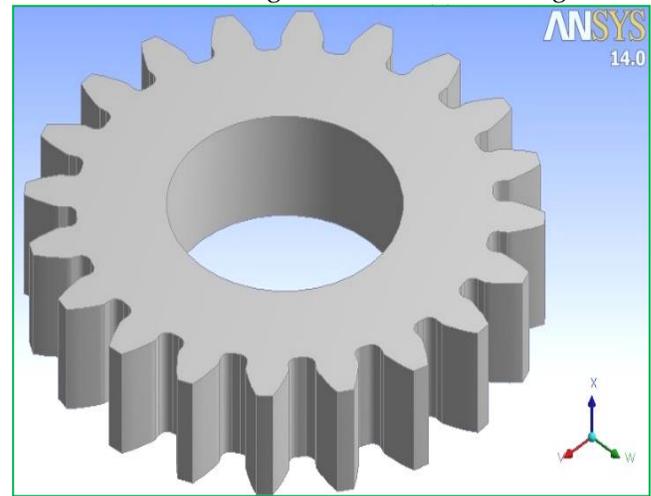


Fig (3) : ANSYS simulated 3D model of spur gear (10)

Static structural preference in ANSYS software for analysis is used. The material properties for the structural considered is steel; the yield strength 250 MPa, ultimate yield strength 460 MPa, the poisson's ratio is 0.3 and modulus of elasticity as 200 GPa. The three-dimensional gear modeled by extruding the two-dimensional geometry and the contact area between two gears are indicated in Fig. 4.

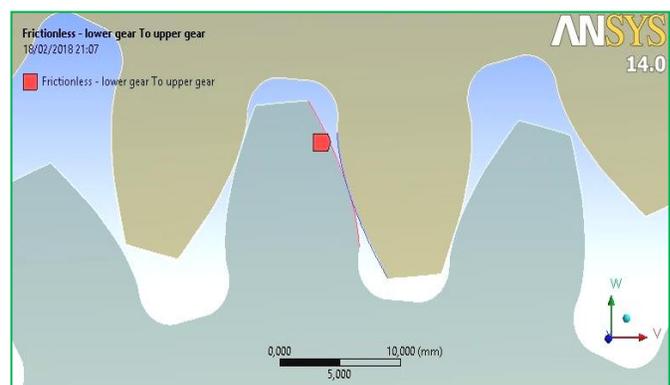


Fig (4) : The contact area between two gears

The 3D gear is meshed and refined the contact region in order to obtain more accurate results. The mesh type used in this research is free mesh because it can give more accurate results when compared with rectangular type. In Fig. 5, the mesh type and more dense area of the mesh are illustrated, see (Y.-K. Khdir, Kanit, Zaïri, & Nait-Abdelaziz, 2014, 2015; Y. Khdir, Kanit, Zaïri, & Nait-Abdelaziz, 2013; Y. K. Khdir, 2016).

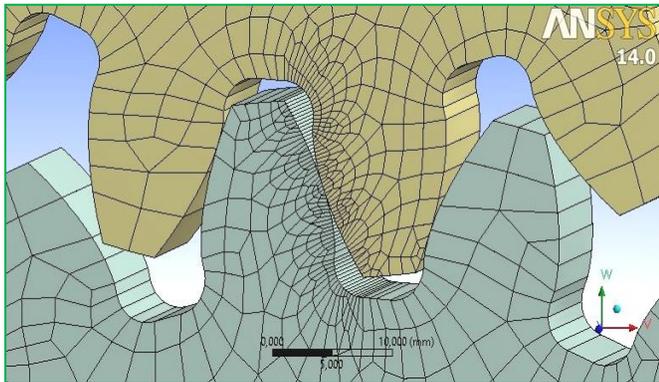
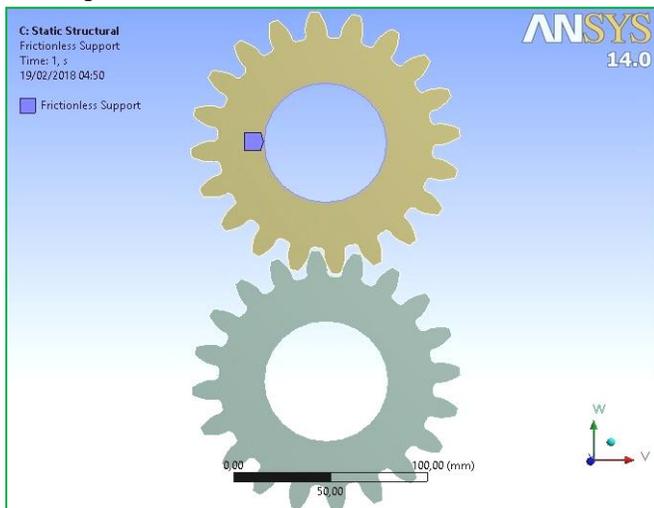


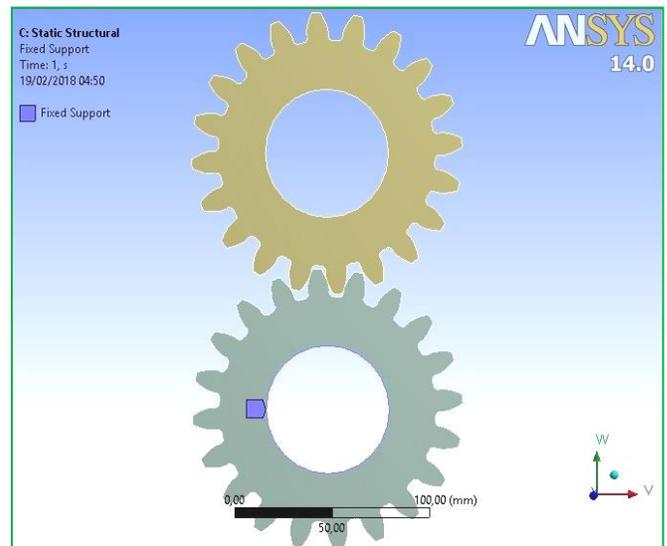
Fig (5) : Meshing technique and the density of mesh of the contact area

#### 4.2. Finite Element Analysis

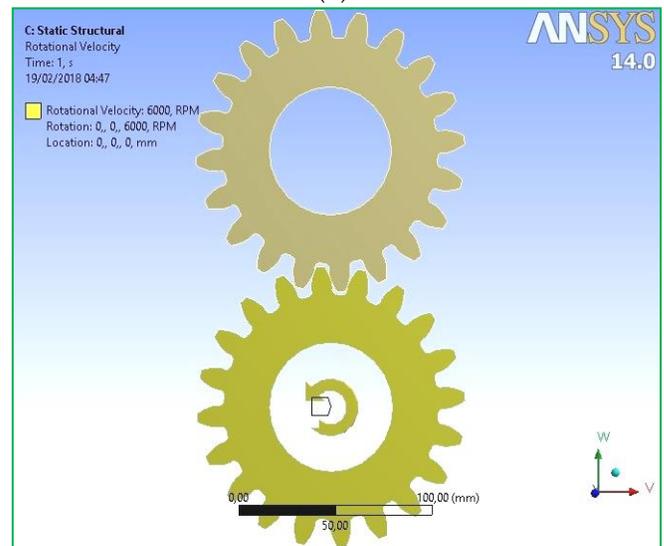
After creating the 3D geometry of the two gears; for the first gear fixed supported is applied and for the second gear frictionless support is considered, see Fig. 6a and 6b. The boundary conditions applied on the two gears are as follows; the various ranges of moments are applied on the inner rim of upper gear in clockwise direction as shown in Fig. 6c and Rotational velocity is applied on the Z-axis to the fixed center of the rotation respectively in various ranges in counterclockwise direction as shown in Fig. 6d. After applying all the conditions in ANSYS, the minimum and maximum equivalent contact stress can be determined between the two gears. Also, the stress distribution over the contact area of the two gears and the local stress at each point can be predicted.



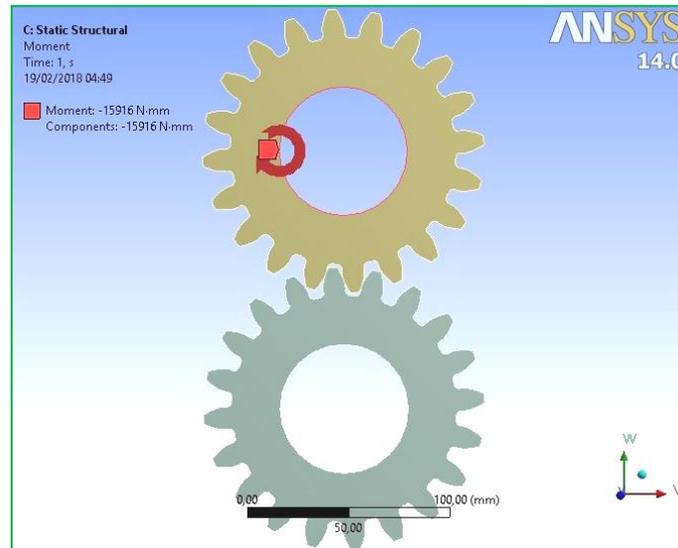
(A)



(B)



(C)



(D)

Fig (6) : The boundary condition of the two gears before running the software  
 (a) Frictionless support (b) Fixed support (c) Moment and (d) Rotational velocity

**5. RESULT AND DISCUSSION**

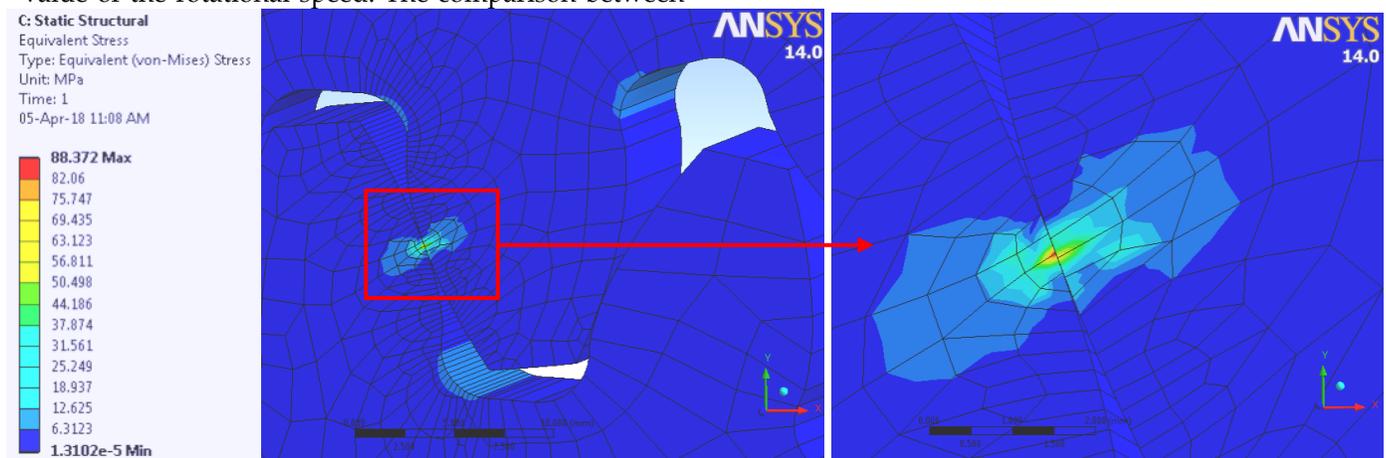
Fig. 7 shows the FE solution of the contact stress between the two gears for 6 different cases. For each case the rotational speed is provided with the applied moment. Fig. 7a shows the first case of the simulation which represents case no. 1, the rotational speed is 3500 rpm, and moment is 27283.7 N.mm. It has noticed that there are different ranges of stress according to the

different colors as illustrated in the figure. Based on the software data, the equivalent contact stress is determined which is equal 88.378 MPa. This result can be compared with the exact results obtained from the Hertzian equation which is equal to 96.79 MPa. Also we can calculate the error percentage according to the below equation:

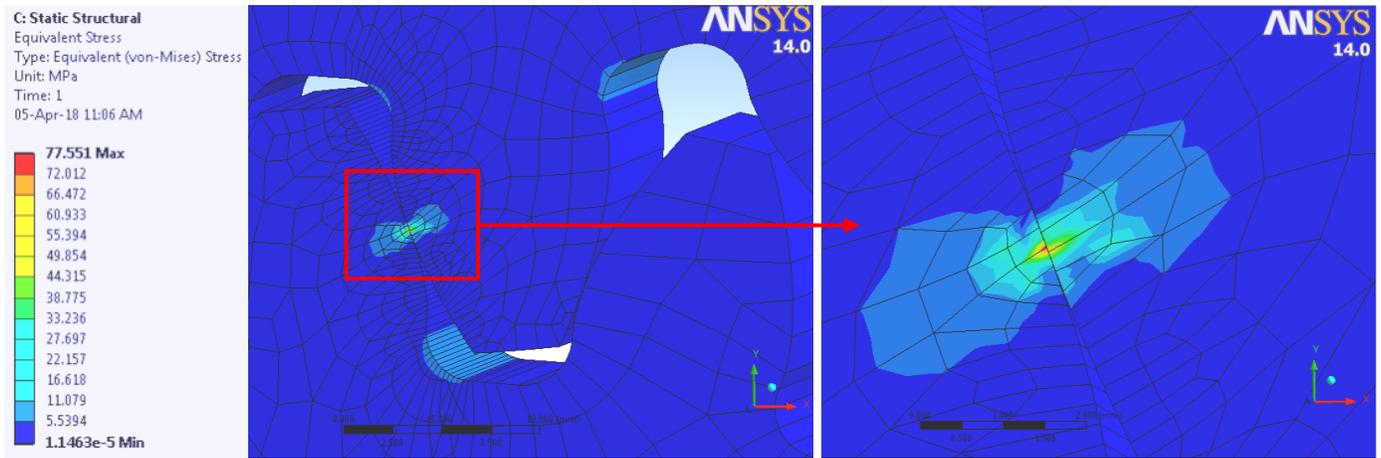
$$Error \% = \frac{\text{Maximum contact stress (FE ANSYS)} - \text{Maximum contact stress (Hertzian)}}{\text{Maximum contact stress (Hertzian)}} \times 100 \tag{11}$$

For the other cases as presented in Fig. 7, there are different ranges of stress distribution according to the value of the rotational speed. The comparison between

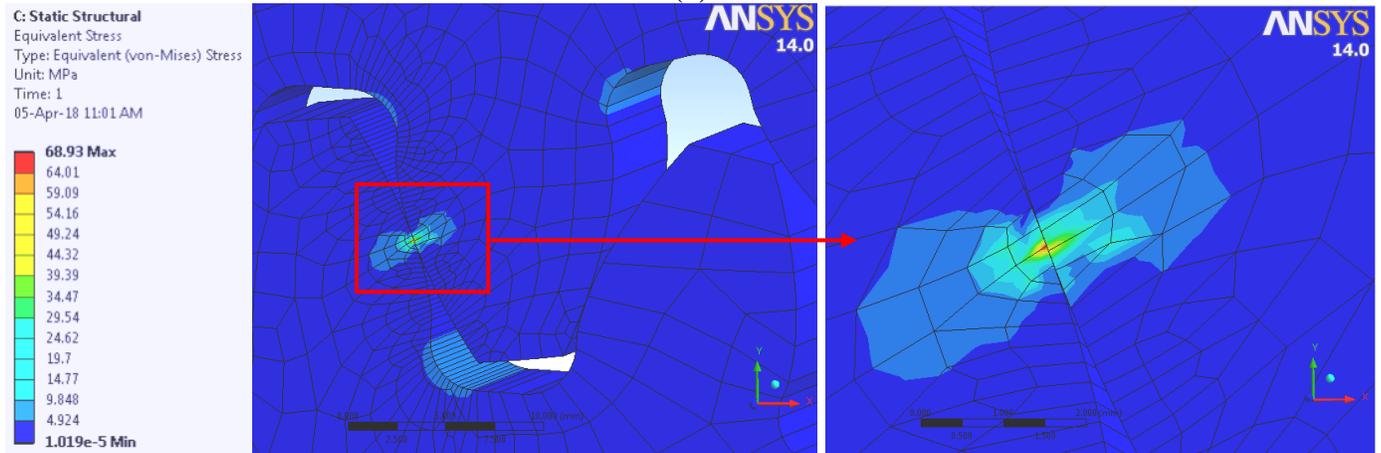
the exact results of Hertzian equation and the FE results using ANSYS software are presented in Table 2.



(a)

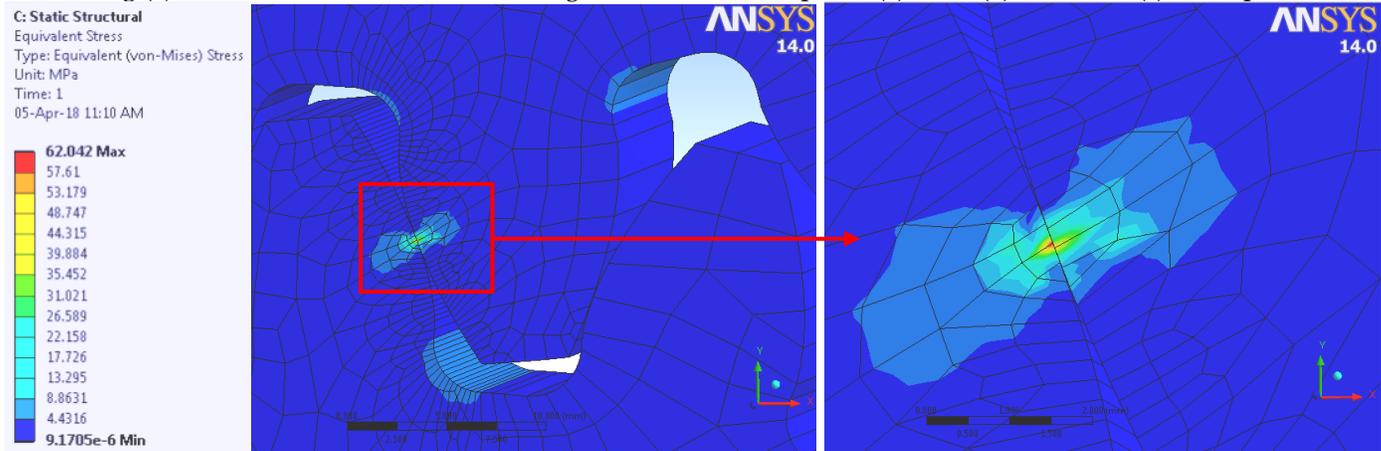


(b)



(c)

Fig (7) : Contact stress between the two gears, for different speeds (a) 3500, (b) 4000, and (c) 4500 rpm



(d)

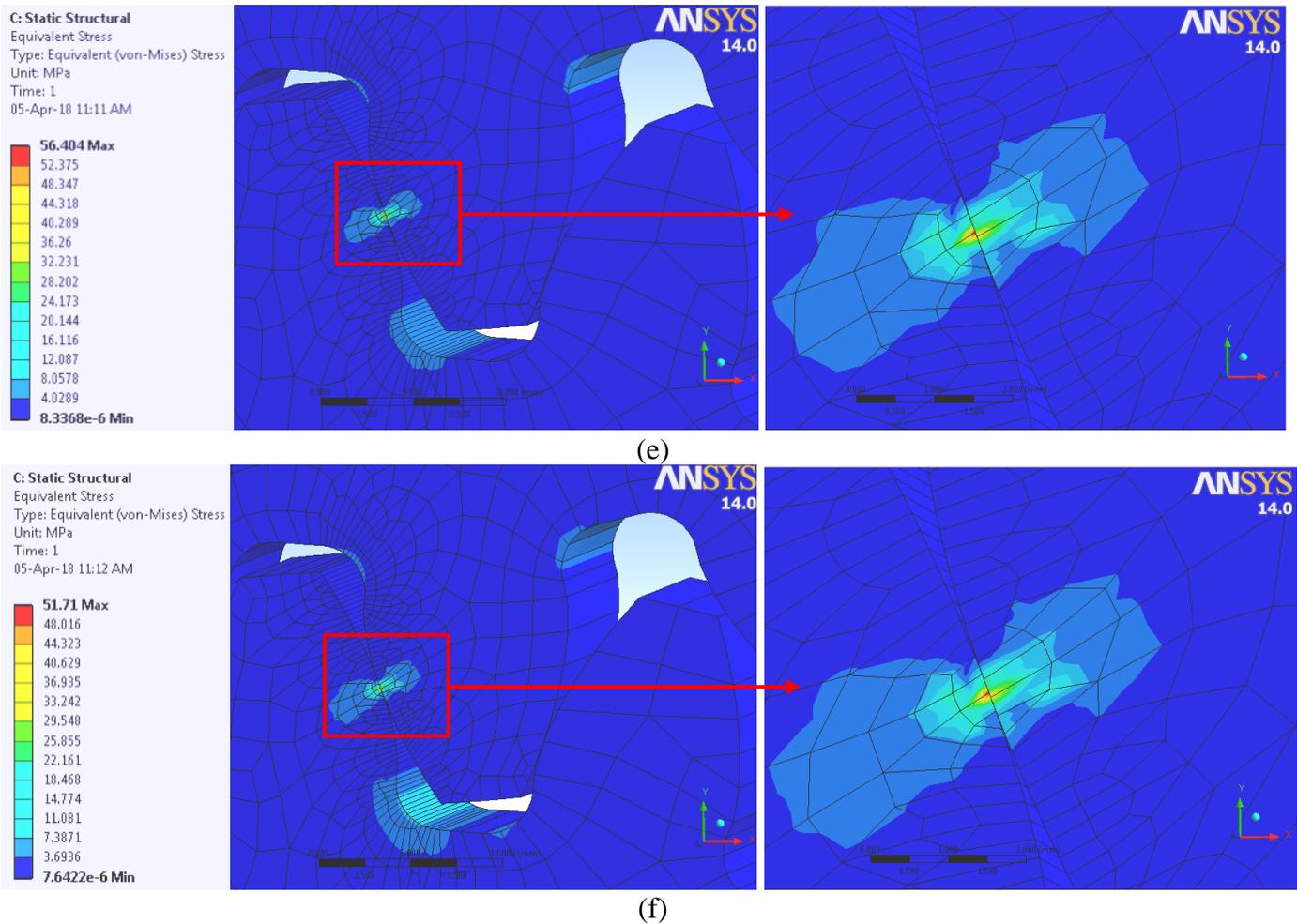


Fig (7) : Contact stress between the two gears, for different speeds, (d) 5000, (e) 5500, and (f) 6000 rpm

TABLE (2) : Maximum contact stress results in ANSYS and Theoretical calculations

No.	Rotational speed(rpm)	Applied moment	Maximum contact stress		Error
			MPa		
	<i>Rpm</i>	<i>Nmm</i>	ANSYS	Hertz	%
1	3500	27283.7	88.378	96.79	8.79
2	4000	23873.24	77.56	79.52	2.465
3	4500	21221	68.896	66.6	3.45
4	5000	19098.6	62.043	56.9	9.04
5	5500	17362.4	56.404	49.32	14.36
6	6000	15915.5	51.708	43.286	19.46

In Fig. 8 the relationship between the contact stress in MPa and the rotational speed (rpm) for the two gears are shown

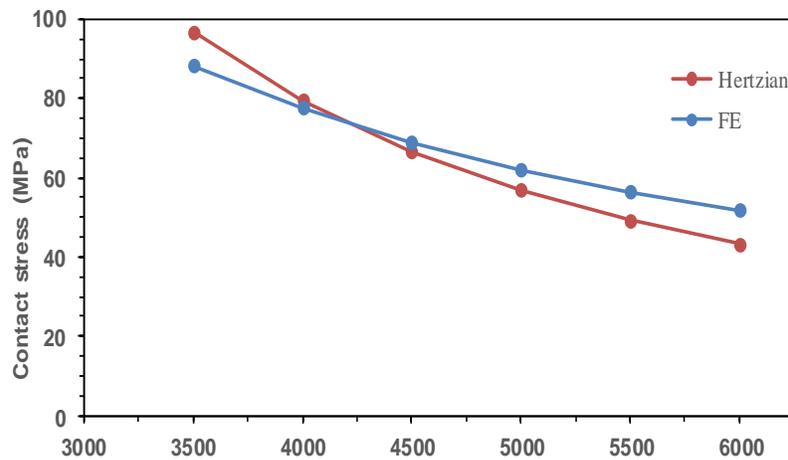


Fig (8) : The relationship between the rotational speed and contact stress

The results of Hertzian equation and finite element analysis for rotational speed obtained by considering different values of (3500 to 6000) rpm as presented in Table 2. The contact stress has been reduced in both methods with increasing rotational speed. When the rotational speed is 3500 rpm the maximum value of contact stress is 96.79 MPa, if we use Hertzian equation comparing to the other cases readings. Whereas, this value is reduced up to 88.378 MPa using FE solution using ANSYS for the same value of speed, with the error percentage of 8.79%. However, there is no significant difference between Hertzian equation FE result when the rotational speed increased from 5000 rpm to 6000 rpm which is the error percentage increased from 9% to 14.36%, and 19.46%. These results show a degree of agreement with the percentage error 2.465% and 3.45 % at 4000 rpm and 4500 rpm that is acceptable.

## 6. CONCLUSION

In this research the following points are concluded:

- The influence of increasing speed on contact stress that is due to increase the vibration of gear teeth and cause pitting is caused by the repetition of the contact between teeth mating, and it leads to the dynamic body force and is more effective.
- There is an acceptable error percentage between exact and FE solution.
- When comparing between the exact and FE results, it is possible to use ANSYS to analyze the effect of the other variables.
- The ANSYS solver is capable to evaluate a very difficult cases, and it is possible to use FE method to study different materials and sizes.
- The location of the maximum and minimum stress can be predicted using FE method.
- According to the data of this research the more acceptable spur gear speed is 4000-4500 rpm.

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