Using queuing theory to solve the problem of momentum to issuing the driving licenses in Dohuk city

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ABSTRACT

Queuing theory is a mathematical study of so-called queues or waiting lines. This phenomenon is common in daily life as in gas stations, airports, repair workshops and other common everyday examples. Waiting occurs when service demand is higher than service system power. Due to the difficulty in predicting the number of customers arriving and the time taken by the customer at the service station, the process of obtaining performance metrics is necessary before the queuing systems are implemented. When the service system power is too high, the system is charged at a high cost. Conversely, when the system power is low (insufficient for the customer service), the waiting time in the queue increases, As well as loss of order to its customers. Therefore, attention has been drawn to the so-called theory of waiting lines to solve such problems to reach a balance in the work of the system.

This research aims to overcome the difficulties experienced by citizens in obtaining market holidays on time and reduce waste in time and the cost of waiting.

The results were as shown in tables (l) to (6) of the city center and according to the distribution of access and service of the model used (G / G / C). We note in the Poisson distribution with exponential that the average number of customers in the system Ls = 5.527), which is approximately (5 customers), which is waiting in the system. We note in the previous distribution itself that the average number of customers in the waiting queue (customer 2.1924 = L q) is approximately 2 (customer) which is waiting in queue. The Poisson distribution with the exponential is that the average time spent by the customer in the system (minute W s = 16.5772). Note in the Poisson distribution with the exponential that the average time spent by the customer in the waiting queue is (minute W = 6.5772) We note in the Poisson distribution with exponential that The average number of customers in the system (customer Ls = 4.3258) is about (4 customers) which is waiting in the system. We see in the previous distribution itself that the average number of customers in the queue (customer 2.0 = L q)) There is no waiting in the queue. The Poisson distribution with exponential is the average time spent by the customer in the system (min W s = 11.3333). We note in the Poisson distribution with exponential that the average time spent by the customer in the waiting queue is (min W q = 4.6666)

Keywords: Poisson distribution, Exponential Distribution. The multiple channel waiting line with finite capacity (M / M / C): (GD / N / ∞). The Multiple channel waiting line Model with Infinite Capacity(M/M/C):(GD/∞/∞). Model queue stations, multi- service and capacity of non-specific (G /G /C): (GD /∞/ ∞)

1. Introduction

In view of the momentum of the windows of issuing market licenses in the city of Dohuk Artat researcher to conduct a field study to stand on the bottlenecks of this congestion and find effective solutions for this congestion and conduct a future study to eliminate this congestion and find practical solutions to alleviate the waste in waiting time.

Queuing theory is a mathematical study of so-called queues or waiting lines. This phenomenon is common in daily life as in gas stations, airports, repair workshops and other common everyday examples. Waiting occurs when service demand is higher than service system power. Due to the difficulty in predicting the number of customers arriving and the time taken by the customer at the service station, the process of obtaining performance metrics is necessary

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before the queuing systems are implemented[1,2,3].

When the service system power is too high, the system is charged at a high cost. Conversely, when the system power is low (insufficient for the customer service), the waiting time in the queue increases, As well as loss of order to its customers. Therefore, attention has been drawn to the so-called theory of waiting lines to solve such problems to reach a balance in the work of the system [1,2,3].

1.2 Objective of the research

This research aims to overcome the difficulties experienced by citizens in obtaining market holidays on time and reduce waste in time and the cost of waiting.

<u>*Theoretical aspect*</u> [4,5,6]: Waiting time in line for service occurs when the demand for services exceeds the available capacity of the plant that provides the services.

In the case of high service system capacity is in place that will reduce time in queue line but also it will be of high cost to the system, on the contrary if the capacity of a service system is low (inadequate customer service), this leads to increase the waiting time in the queue line causing high waiting cost as well loss of customers, to resolve these problems a balance in the service system should be reached by implementing the so-called *Queuing theory*.

Queuing theory is a mathematical procedure to solve and deal with the issue of waiting in queue lines, so in order to apply the queuing theory it is necessary to determine the practical performance measures in the service system *(measures of performance)*. Queuing theory is not a method to optimize service system efficiency but to make decisions in turning the process.

Data of this research have been made of the following distributions:

- *Poisson distribution*:The discrete probability distribution is used to describe the arrival of random customers to the system queue subject that the following conditions are true:
    - Orderliness: intended in any period of time up at most one customer arrives to the service station.
    - Stationarity: that is, within a certain time be the possibility of the arrival of a customer during a certain period of time is the same for all of the term equal periods of time.
    - Independence: means that customers arrive independently from one another up does not affect any access in a particular period of time on the possibility of access in other periods of time.
    - After verifying the above conditions in the queue model, then probability of number of customers' arrival (n) in the period (t) can be written in the following formula:

$$P_{(n)}(t) = \frac{(\lambda t)^n e^{-\lambda t}}{n!} \quad , \quad n = 0,1,2,\ldots \quad , \quad \lambda > 0 \quad \ldots.. (1)$$

where: λ : arrival rate per unit time .

t : time interval length.

n : number of customers arrival

- *Exponential Distribution:* The continuous probability distribution represents random time of customer service in the queue system. The probability density function*(p. d. f)* to service time *(t)* is:

$$f(t) = \mu e^{-\mu t} \quad ; \quad t > 0 \quad \ldots\ldots\ldots\ldots\ldots (2)$$

where μ : rate service per unit time.

t : interval time length.

2. The multiple channel waiting line with finite capacity (M / M / C): (GD / N / ∞)[6,7]

This model is characterized by the presence of *(C)* of the service stations and that the system capacity which is also defined a maximum of *(N)*customers and this capacity should be the number of service stations at least *(C ≤ N)*, although the maximum length of waiting

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line is *(N - C)*. Therefore, any additional customer will be rejected in the case of reaching the number of customers in the system *(N)*, and it is access *(λ)* equal to zero in this case

$$\lambda_n = \begin{cases} \lambda \;\; ; \;\; n = 0,1,2,\dots,N-1 \\ 0 \;\; ; \;\; n = N, N+1, \dots \dots \dots \end{cases} \quad \dots\dots(3)$$

As for the average service time *(μ)* shall be:

$$\mu_n = \begin{cases} n\mu \;\; ; \;\; 0 \le C \le N \\ C\mu \;\; ; \;\; C \le n \le N \end{cases} \dots\dots\dots\dots\dots\dots(4)$$

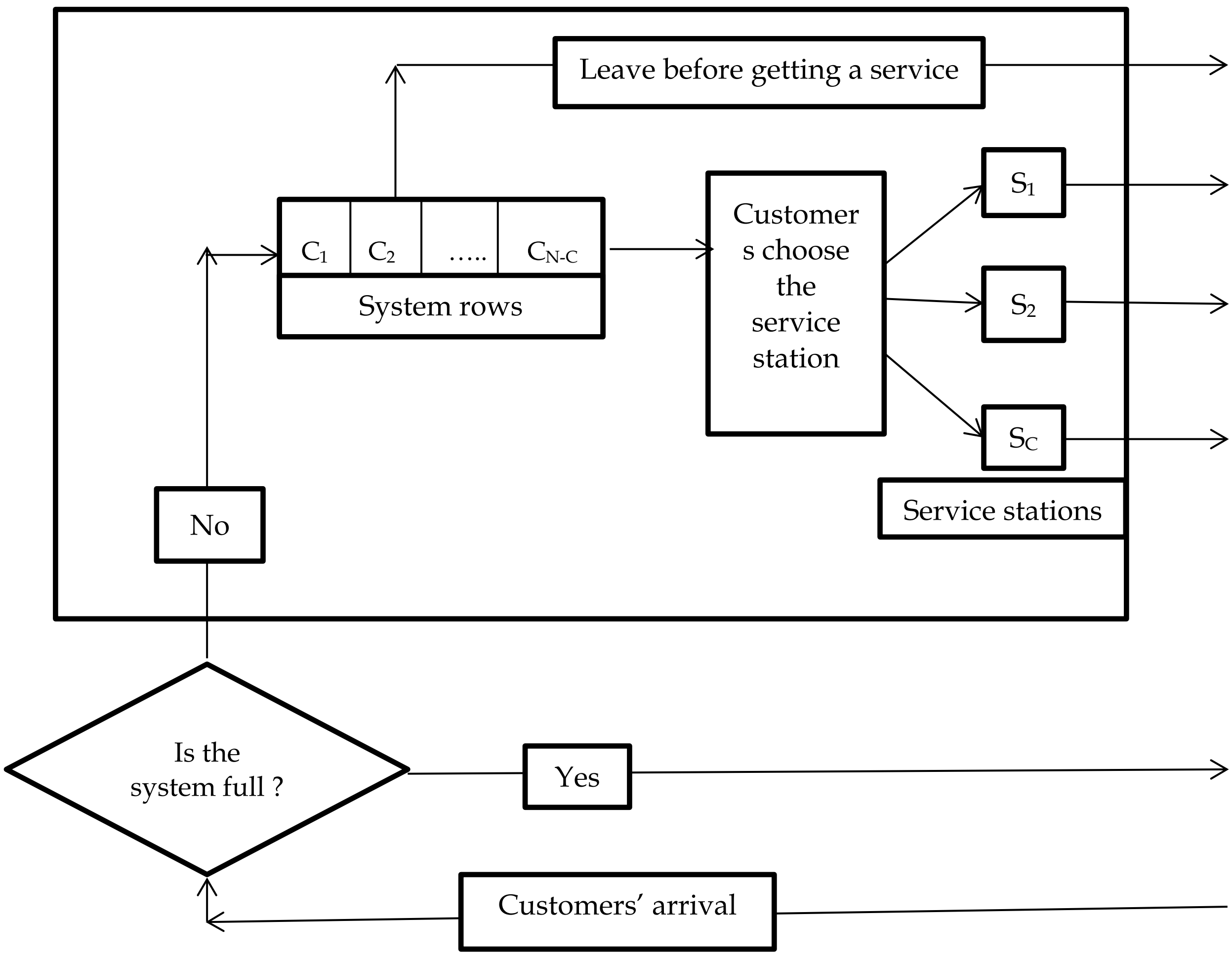

Figure (1) represents the queue type (M / M / C)

In a stable situation, it is possible to transit operations for the system as shown in figure (2).

The mathematical equations for calculating the efficiency of the model performance measures is:-

The probability of no any customers in the system *(P_0)*:

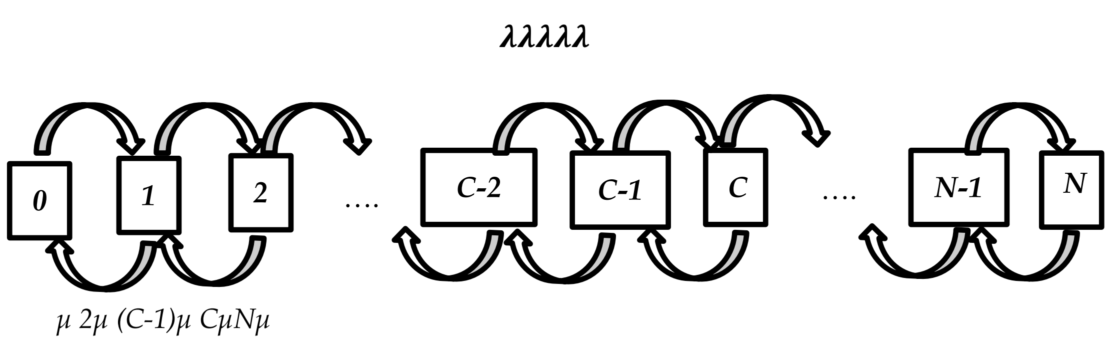

Figure (2) represents operations possible transition to the model

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$$P_0 = \begin{Bmatrix} \left[\sum_{n=0}^{C-1}\frac{\rho^n}{n!} + \frac{\rho^C\left(1-\left(\frac{\rho}{C}\right)^{N-C+1}\right)}{C!(1-\frac{\rho}{C})}\right]^{-1} & \frac{\rho}{C} \neq 1 \\ \left[\sum_{n=0}^{C-1}\frac{\rho^n}{n!} + \frac{(N-C+1)\rho^C}{C!}\right]^{-1} & \frac{\rho}{C} = 1 \end{Bmatrix} \quad \ldots\ldots (5)$$

Andt he probability of a *(n)* customers in the system *(P_n)* :

$$P_n = \begin{Bmatrix} \frac{\rho^n P_0}{n!} & 0 \le n < C \\ \frac{\rho^n}{C!C^{n-C}} P_0 & C \le n < N \end{Bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots(6)$$

Also the average number of customers in the queue *(L_q)*:

$$L_q = \begin{Bmatrix} P_0\frac{\rho^{C+1}}{(C-1)!(C-\rho)^2}\left[1-\left(\frac{\rho}{C}\right)^{N-C} - (N-C)\left(\frac{\rho}{C}\right)^{N-C}\left(1-\frac{\rho}{C}\right)\right] & \frac{\rho}{C} \neq 1 \\ P_0\frac{\rho^C(N-C)(N-C+1)}{2C!} & \frac{\rho}{C} = 1 \end{Bmatrix}$$

….. (7)

And The average number of customers in the system *(L_s)*:

$$L_s = L_q + (C - \bar{C}) = L_q + \frac{\lambda_{eff}}{\mu} \quad \ldots\ldots\ldots\ldots..(8)$$

For them one the Average waiting time in the system *(W_s)*:

$$W_s = \frac{L_s}{\lambda_{eff}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(9)$$

And the Average waiting time in the queue *(W_q)*:

$$W_q = \frac{L_q}{\lambda_{eff}} \quad \ldots\ldots\ldots\ \ldots\ldots\ldots(10)$$

And hence effective arrival rate:

$$\lambda_{eff} = \lambda(1 - P_n) = \mu(C - \bar{C}) \ldots\ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Finally the expected number of empty service stations:

$$\bar{C} = \sum_{n=0}^{C}(C-n)P_n \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ \ldots\ldots\ldots(12)$$

3-The Multiple channel waiting line Model with Infinite Capacity(M/M/C):(GD/∞/∞)[8,9]:

Thedistributionof arrival to this system follows a Poisson distribution at a constant rate is ,the distribution of service time follows the exponential distribution at a constant rate (1/μ) for all units. The method of providing the service and the system accommod atestheunits (the length of the queue) is an unlimited , and the source of units is also unlimited .In order to get these results we have the following formulas:

$$\lambda_n = \lambda \qquad for\ all\ n \ge 0$$

$$\mu_n = \begin{Bmatrix} n\mu & n \le C \\ C\mu & n \ge C \end{Bmatrix} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots...(13)$$

The probability of a *(n)* customers in the system *(P_n)* :

$$P_n = \begin{Bmatrix} \frac{\lambda^n}{n!\mu^n}P_0 & n \le C \\ \frac{\lambda^n}{C!C^{n-C}\mu^n}P_0 & n \ge C \end{Bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (14)$$

And the value of P_0, can be identified through the fact that the sum of all possible probabilities is equal to one:

$$\sum_{n=0}^{\infty} P_n = 1 = P_0\left[\sum_{n=0}^{C-1}\frac{\rho^n}{n!} + \sum_{n=C}^{\infty}\frac{\rho^n}{C!C^{n-c}}\right] \Rightarrow P_0 = \left[\sum_{n=0}^{C-1}\frac{\rho^n}{n!} + \sum_{n=C}^{\infty}\frac{\rho^n}{C!C^{n-C}}\right]^{-1} \ldots(15)$$

Taking $j = n - C$, then :

$$P_0 = \left[\sum_{n=0}^{C-1}\frac{\rho^n}{n!} + \frac{\rho^C}{C!\left(1-\frac{\rho}{C}\right)}\right]^{-1} \quad ;\ \frac{\rho}{C} < 1 \ \ldots\ldots\ldots..(16)$$

Then the probability of a (n) of customers in the system *(P_n)* can be writhen as:-

$$P_n = \begin{Bmatrix} \frac{\rho^n}{n!}P_0 & 0 \le n \le C \\ \frac{\rho^n}{C^{n-C}C!}P_0 & n > C \end{Bmatrix} \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(17)$$

Now, the average number of customers in the queue *(L_q)* is:-

$$L_q = \frac{\rho^{C+1}P_0}{(C-1)(C-\rho)^2} = \left[\frac{C\rho}{(C-\rho)^2}\right]P_C \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (18)$$

And also the average number of customers in the system *(L_s)*:

$$L_s = L_q + \rho \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots..........(19)$$

Hence the Average waiting time in the system *(W_s)*:

$$W_s = W_q + \frac{1}{\mu} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(20)$$

And the average waiting time in the queue *(W_q)*:

$$W_q = \frac{L_q}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots..(21)$$

where:

$$\rho < C \quad ; \quad \lambda < \mu C$$

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4. Model queue stations, multi- service and capacity of non-specific (G /G /C): (GD /∞/ ∞)[8,9,10]:

This model differs from the previous model the fact that two-way arrival times and service times may take any distribution of statistical distributions as the Normal distribution , Gamma distribution , Log Normal distribution , Weibull Distribution , Exponential distribution or any other distribution .

The waiting processes for this model , must be verify the following:

- The number of arrival units and the servicetimes distributed Gamma.
- There are *C* of the customer service stations.
- There is no limit to how accommodating the waiting system
- There are no limits of population which customers comes from.
- General service pattern is unlimited.

And hence:

The average number of customers in the queue *(L_q)*:

$$L_q = P_0 \frac{\rho^{C+1}}{C!C} \cdot \frac{1}{\left(1-\frac{\rho}{C}\right)^2} \cdot \frac{\mu^2 V(t) + V(t')\lambda^2}{2} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (22)$$

where :

$V(t)$ *:* Variation of service times.

$V(t')$ *:* Variation ofarrival times.

$\rho\mu = \lambda$, $\rho c \prec 1$

Assenting the average number of customers in the system *(L_s)* is:

$$L_s = L_q + \rho \quad \ldots\ldots\ldots\ldots\ldots\ldots \ (23)$$

And the Average waiting time in the queue *(W_q) is:*

$$W_q = \frac{L_q}{\lambda} \quad \ldots\ldots\ldots\ldots\ldots \quad (24)$$

Also the Average waiting time in the system *(W_s)is:*

$$W_s = W_q + \frac{1}{\mu} \quad \ldots\ldots\ldots\ \ldots\ldots.(25)$$

Finally the probability of a*(n)* of customers in the system P_n *:*

$$P_{(n)} = \begin{Bmatrix} P_{(n-1)} \frac{\rho}{n} & for\ n \le C \\ P_{(n-1)} \frac{\rho}{C} & for\ n > C \end{Bmatrix} \quad \ldots\ldots\ldots(26)$$

5. Application side

The research aims, as mentioned above, to use the queuing theory to solve the problem of the driving license issue in Dohuk city, as applied to four windows to issue the market licenses in the city of Medina to the momentum achieved in order to find the appropriate mathematical model. Data on the four field windows were collected for three days each and from 9 am to 1 pm (for 4 hours per day). The time period is calculated from equation (27).

$$\Delta t = \frac{h}{C} * 60 \quad \ldots\ \ldots\ldots\ldots\ (27)$$

where :

h: Representing the actual number of hours worked.

c: Representing the number of citizens in the system.

The arrival data based on the number of customers arriving during the time period Δt , by using the software programming ((*stat graphics*)) show that the Poisson distribution is the appropriate distribution of arrival data . Service time the difference between the time of starting and termination the service, and after collection the data of service times for each desk of the four desks , by application of software ((*stat graphics*)) we conclude that the exponential distribution is the appropriate distribution to times of service.

Thus the statistical results of the distribution (Poisson distribution for the number of arrivals and exponential distribution for service time) are shown as follow:

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Arrival distribution		*Service distribution*	
Poisson	*Mean = 3* *D= 281. 384* *P – Value <* *0.01*	*Exponential*	*Mean 10* *D = 0.411353* *P – Value <* *0.01*

The use of software- program ((*Win QSB*)) , to solve the problem of momentum of driving licence issuance at the four service desks, we got the best results as in the following table :

Table (1) entry data of the program Win QSB (Poisson and Exponential distribution) for 4 desks

Data Description	ENTRY
Number of servers	4
Service time distribution (in minute)	Exponential
Location parameter (a)	
Scale parameter (b>0) (b=mean if a=0)	10
(Not used)	
Service pressure coefficient	
Interarrival time distribution (in minute)	Poisson
Mean (u)	3
(Not used)	
(Not used)	
Arrival discourage coefficient	
Batch (bulk) size distribution	Constant
Constant value	1
(Not used)	
(Not used)	
Queue capacity (maximum waiting space)	M
Customer population	M
Busy server cost per minute	
Idle server cost per minute	
Customer waiting cost per minute	
Customer being served cost per minute	
Cost of customer being balked	
Unit queue capacity cost	

Table (2) the results data of the program Win QSB (Poisson and Exponential distribution) for 4 desks

01-30-2016	Performance Measure	Result
1	System: G/M/4	From Approximation
2	Customer arrival rate (lambda) per minute =	0.3333
3	Service rate per server (mu) per minute =	0.1000
4	Overall system effective arrival rate per minute =	0.3333
5	Overall system effective service rate per minute =	0.3333
6	Overall system utilization =	83.3333 %
7	Average number of customers in the system (L) =	5.5257
8	Average number of customers in the queue (Lq) =	2.1924
9	Average number of customers in the queue for a busy system (Lb) =	3.3333
10	Average time customer spends in the system (W) =	16.5772 minutes
11	Average time customer spends in the queue (Wq) =	6.5772 minutes
12	Average time customer spends in the queue for a busy system (Wb) =	10.0000 minutes
13	The probability that all servers are idle (Po) =	2.1310 %
14	The probability an arriving customer waits (Pw) or system is busy (Pb) =	65.7722 %
15	Average number of customers being balked per minute =	0
16	Total cost of busy server per minute =	$0
17	Total cost of idle server per minute =	$0
18	Total cost of customer waiting per minute =	$0
19	Total cost of customer being served per minute =	$0
20	Total cost of customer being balked per minute =	$0
21	Total queue space cost per minute =	$0
22	Total system cost per minute =	$0

Table (3) entry data of the program Win QSB (Poisson and Exponential distribution) for 5 desks

Data Description	ENTRY
Number of servers	5
Service time distribution (in minute)	Exponential
Scale parameter (b>0) (b=mean if a=0)	10
Inter arrival time distribution(in minute)	Poisson
Mean (u)	3

Table (4) the results data of the program Win QSB (Poisson and Exponential distribution) for 5 desks

Performance Measure	*Result*
System G/M/5	*From approximation*
Average number of customers in the system (L_S)	*4.3258*
Average number of customers in the queue (Lq)	*2.0*
Average time customers spends in the system (W_S)	*11.3333 minutes*
Average time customers spends in the queue (Wq)	*4.6666 minutes*

Table (5) entry data of the program Win QSB (Poisson and Exponential distribution) for 6 desks

Data Description	*ENTRY*
Number ofservers	*6*
Service time distribution (in minute)	*Exponential*
Scale parameter (b>0) (b=mean if a=0)	*10*
Interarrival time distribution(in minute)	*Poisson*
Mean (u)	*3*

Table (6) the results data of the program Win QSB (Poisson and Exponential distribution) for 6 desks

Performance Measure	*Result*
System G/M/6	*From approximation*
Average number of customers in the system (L_S)	*2.1222*
Average number of customers in the queue (Lq)	*1.3*
Average time customers spends in the system (W_S)	*6.2223 minutes*
Average time customers spends in the queue (Wq)	*2.3435 minutes*

6. Conclusions

Using *(G / G / C)* model, Poisson distribution for arrival of random customers to the system queue and the exponential distribution for random time of customer service in the queue system .From tables (1) - (6) ,we conclude that :

- The rate number of customers in the system of 4 desks is $Ls = 5.5257$ or approximately *6* customers (table *2)*, there is waiting in the system. This rate number of customers decrease to *4.3258* or approximately *4* customers at *5* desks (table *4*), also decrease to *2.12* or approximately *2* customers at *6* desks (table *6*).
- The rate number of customers in the queue L_q = *2.1924* or approximately *2* customer (table *2*), there is waiting in queue. This rate number of customers decrease to *2* at *5* desks (table *4*), also decrease to *1.3* or approximately *1* customer at *6* desks (table*6*).
- The rate time spent by the customer in the system is W_s= *16.5772* minutes at 4 desks (table *2)*. This rate time decrease to *11.3333* minutes at *5* desks (table *4*) , also decrease to *6.2223* minutes at *6* desks (table *6*).
- The rate time spent by the customer in the queue is W_q = *6.5772* minutes at 4 desks (t. table *2)*. This rate time decrease to *4.6666* minutes at *5* desks (table *4*) , also decrease to *2.3435* minutes at *6* desks (table *6*).
- When adding two desks for customer service (from *4* to *6*desks) , we note that:
  - The average number of customers in the system decrease from *6* to *2* customers.
  - The average number of customers in the queue decreased from *2* to *1*customers.
  - The rate time spent by the customer in the system decreased from *16.6* to *6.2* minutes.
  - The rate time spent by the customer in the queue decreased from *6.5* to *2.3* minutes.
- Through the current reality in the region of stability and stability of security has been a large demand for market licenses market so we see queues of people waiting
- The momentum of the windows of the issuance due to lack of cadres.
- 8As we have previously mentioned the stability of the security region, there has been a lot of citizens buy cars, so increased demand for market licenses

7. Recommendations

- Licence department should have database for its activities to enable
- researchers to conduct developmental researches for future studies.
- Rehabilitation of dedicated staff in the saved data on the computers so

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- that they can fulfil customer transactions.
- Find a certain criteria to calculate the waited time.
- Find the financial cost of wasting time.
- Increase the number of desks of driving licence issuance Dohuk toreduce long queue and waiting time.

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