Some Properties of Soft Delta-Topology

1Ramadhan A. Mohammed, 2O. R. Sayed, 3A. Eliow

1Department of Mathematics, College of Basic Education, University of Duhok, Kurdistan-Region – Iraq
2Department of Mathematics, Faculty of Science, Assiut University, Assiut-Egypt
3Department of Mathematics, Faculty of Science, Sohag University, Sohag-Egypt

ABSTRACT

In this paper, we apply the concept of soft sets to δ-open set and δ-closed set. The associated soft δ-topology in terms of soft δ-open sets were introduced and some properties of them were investigated. Moreover, the definitions, characterizations and basic results concerning soft δ-interior, soft δ-closure, soft δ-boundary and soft δ-exterior were given. Finally, the concept of soft pu-δ-continuity was defined and some properties of it were introduced.

Keywords: Soft set; soft δ-open set; soft δ-topology; soft pu-δ-continuity.

1. Introduction

Some concepts in mathematics can be considered as mathematical tools for dealing with uncertainties, namely theory of vague sets, theory of rough sets and etc. But all of these theories have their own difficulties. The concept of soft sets was first introduced by Molodtsov as a general mathematical tool for dealing with uncertain objects (Dmitriy Molodtsov, 1999). He successfully applied the soft theory in several directions, such as smoothness of functions, game theory, probability, Perron integration, Riemann integration, theory of measurement (Dmitriy Molodtsov, 1999; DA Molodtsov, 2001; D Molodtsov, 2004; D Molodtsov, Leonov, & Kovkov, 2006). It is remarkable that, Molodtsov used this concept in order to solve complicated problems in other sciences such as, engineering, economics and etc. The soft set theory has been applied to many different fields. Later, few researches((Aygünoğlu & Aygün, 2012),(Çağman, Karataş, & Enginoğlu, 2011),(Georgiou & Megaritis, 2014),(Hussain & Ahmad, 2011),(Min, 2011),(Peyghan, Samadi, & Tayebi),(Shabir & Naz, 2011),(Zorlutuna, Akdag, Min, & Atmaca, 2012)) introduced and studied the notion of soft topological spaces. Recently, in (2014), S. Yüksel, N. Tozlou and Z. G. Ergül (Ergül, Yüksel, & Tozlou, 2014) initiated soft regular open set and soft regular closed set.

2. Preliminaries

We present here the basic definitions and results related to soft set theory that will be needed in the sequel.

Definition 2.1.(Dmitriy Molodtsov, 1999) Let X to be as an initial universe and E as set of parameters, and P(X) be denoted as the power set of X. A pair (F,A) is referred to as a soft set over X, where F is a mapping given by F:A → P(X) where and A ⊆ E. On the other hand, a soft set over X is a parameterized family of subsets of the universe X, for ε ∈ A, F(ε) might be recognized as the set of ε—approximate elements of the soft set (F,A). The set of all these soft sets over X denoted by SS(X)_{A}.

Definition 2.2.(Maji, Biswas, & Roy, 2003). Let(F,A),(G,B) ∈ SS(X)_{E}. Then (F,A) is a soft subset of (G,B), denoted by (F,A) ⊆ (G,B), if (i) A ⊆ B, (ii) F(ε) ⊆ G(ε), ∀ε ∈ A. In this case, (F,A) is said to be a soft subset of (G,B) and (G,B) is said to be a soft superset of (F,A).

Definition 2.3.(Maji et al., 2003). Two soft subsets (F,A)
and \((G,B)\) over a common universe set \(X\) are said to be soft equal if \((F,A)\) is a soft subset of \((G,B)\) and \((G,B)\) is a soft subset of \((F,A)\).

**Definition 2.4.** (Ali, Feng, Liu, Min, & Shabir, 2009). The complement of a soft set \((F,A)\), denoted by \((F,A)^c\), is defined by \((F,A)^c = (F^c,A)\), \(F^c: A \rightarrow P(X)\) is a function given by \(F^c(e) = X - F(e), \forall e \in A\). Clearly \(((F,A)^c)^c\) is the same as \((F,A)\).

**Definition 2.5.** (Maji et al., 2003). A soft set \((F,E)\) over \(X\) is said to be a null soft set, denoted by \(0_E\) if \(F(e) = \emptyset, \forall e \in A\). A soft set \((F,E)\) over \(X\) is said to be an absolute soft set, denoted by \(1_E\) if \(F(e) = X, \forall e \in E\).

**Definition 2.6.** (Zorlutuna et al., 2012). The soft set \((F,A) \in SS(X)_E\) is called a soft point in \(X\), denoted by \(e_F\), if for the element \(e \in E, F(e) \neq 0_E\) and \(F(e') = 0_E\forall e' \in E \setminus \{e\}\). The set of all soft points of \(X\) is denoted by \(SP(X)\). The soft point \(e_F\) is said to be in the soft set \((G,E)\), denoted by \(e_F \in (G,E)\), if for the element \(e_F \in E, F(e) \subseteq G(e)\).

**Definition 2.7.** (Peyghan et al.; Shabir & Naz, 2011). Let \(X\) be an initial universe set, \(E\) a set of parameters and \(\tau \subseteq SS(X)_E\). We say that the family \(\tau\) a soft topology on \(X\) if the following axioms are true.

1. \(0_E, 1_E \in \tau\).
2. If \((G,E), (H,E) \in \tau\), then \((G,E) \cap (H,E) \in \tau\).
3. If \((G_i,E) \in \tau\) for every \(i \in I\), then \(\bigcup\{G_i,E\}: i \in I\) \(\in \tau\). The triplet \((X,\tau, E)\) is called a soft topological space. The members of \(\tau\) are called soft open sets in \(X\). Also, a soft set \((F,E)\) is called soft closed set if the complement \((F^c,E)\) belongs to \(\tau\). The family of all soft closed sets is denoted by \(\tau^c\).

**Definition 2.8.** (Zorlutuna et al., 2012). Let \((X,\tau, E)\) be a soft topological space and \((F,E) \in SS(X)_E\). A soft topology \(\tau_{(F,E)} = \{(G,E) \cap (F,E) : (G,E) \in \tau\}\) is called a soft relative topology of \(\tau\) on \((F,E)\), and \(\{(F,E), \tau_{(F,E)}\}\) is called a soft subspace of \((X,\tau, E)\).

**Definition 2.9.** (Zorlutuna et al., 2012). A soft set \((G,E)\) in a soft topological space \((X,\tau,E)\) is called

(i) a soft neighborhoods of a soft point \(F(e) \in (X,\tau,E)\) if there exists a soft open set \((H,E)\) such that \(F(e) \in (H,E) \subseteq (G,E)\).

(ii) a soft neighborhood of a soft set \((F,E)\) if there exists a soft open set \((H,E)\) such that \((F,E) \subseteq (H,E) \subseteq (G,E)\).

The neighborhood system of a soft point \(F(e)\) denoted by \(N_e(F(e))\), is the family of all its neighborhood.

**Definition 2.11.** (Peyghan et al.; Shabir & Naz, 2011), (Zorlutuna et al., 2012). Let \((X,\tau, E)\) be a soft topological space and \((F,E) \in SS(X)_E\). The soft closure of \((F,E)\) \([24]\) is the soft set \(Cl(F,E) = \{S,E): (S,E) \in \tau^c, (F,E) \subseteq (S,E)\}\).

The soft interior of \((F,E)\) \([29]\) is the soft set \(Int_s(F,E) = \{S,E): (S,E) \in \tau, (F,E) \subseteq (S,E)\}\).

**Theorem 3.12.** (Zorlutuna et al., 2012). Let \((X,\tau, E)\) be a soft topological space and \((F,E),(G,E) \in SS(X)_E\). Then

1. \((Cl_s(F,E))^c = Int_s(G^c,E)\).
2. \((Int_s(G,E))^c = Cl_s(G^c,E)\).

**Definition 2.13.** (Ergül et al., 2014). Let \((X,\tau, E)\) be soft topological space and \((F,E)\) be a soft set over \(X\):

1. \((F,E)\) is said to be a soft regular open set in \(X\) if \((F,E) = Int_s(Cl_s(F,E))\), denoted by \((F,E) \in SRO(X,E)\).
2. \((F,E)\) is said to be a soft regular closed set in \(X\) if \((F,E) = Cl_s(Int_s(F,E))\), denoted by \((F,E) \in SRC(X,E)\).

**Remark 2.14.** (Ergül et al., 2014). Every soft regular open set in soft topological space \((X,\tau, E)\) is soft open set.

**Definition 2.15.** (Zorlutuna et al., 2012). Let \((X,\tau, E)\) and \((Y,\rho,H)\) be soft topological spaces. Let \(\mu:X \rightarrow Y\) and \(p:E \rightarrow H\) be functions. Then the function \(f_{\mu,p}: SS(X)_E \rightarrow SS(Y)_H\) is defined by:

1. Let \((F,E) \in SS(X)_E\). The image of \((F,E)\) under \(f_{\mu,p}\), written \(f_{\mu,p}(F,E) = ((f_{\mu,p}F),p(E))\) is a soft set in \(SS(Y)_H\) such that

\[
\{x \in p^{-1}(y) \cap A : (f_{\mu,p}F) \cap A \neq \emptyset\}
\]

for all \(\emptyset,\) otherwise.

\[doi: 10.25007/ajnu.v8n4a481\]
Let \( (G,H) \in SS(Y)_H \). The inverse image of \((G,H)\) under \( f_{p\mu}^- \) written as \( f_{p\mu}^- (G,H) = (f_{p\mu}^- (G),p(H)) \) is soft set in \( SS(X)_E \) such that
\[
f_{p\mu}^- (G) = \begin{cases} 
\mu^{-1}(G(p(x))), & \text{for all } x \in E. \\
\emptyset, & \text{otherwise}
\end{cases}
\]

3. Soft regular open set and soft regular closed set

In this section, we introduce some properties of the soft regular open set and soft regular closed set.

**Theorem 3.1.** Let \( (X,\tau,E) \) be a soft topological space. Let \( \{(F_i,E),i \in I\} \in SRO(X,E) \).

1. \((F_1,E) \cap (F_2,E) \in SRO(X,E)\).
2. \((F_1,E) \cup (F_2,E) \notin SRO(X,E)\).

**Proof.** (1) Let \((F_1,E),(F_2,E) \in SRO(X,E)\). Then \((F_1,E) \subseteq Cl_s(F_1,E), (F_2,E) \subseteq Cl_s(F_2,E), (F_1,E) \cap (F_2,E)\) is soft open set. We have \(Int_s((F_1,E) \cap (F_2,E)) = Int_s(F_1,E) \cap Int_s(F_2,E)\). Then \(Int_s((F_1,E) \cap (F_2,E)) \subseteq Int_s Cl_s((F_1,E) \cap Int_s(F_2,E))\), therefore \((F_1,E) \cap (F_2,E)\) \(\subseteq Int_s Cl_s(F_1,E) \cap Int_s(F_2,E)\). Conversely, pick \(e_F \in Int_s Cl_s((F_1,E) \cap Int_s(F_2,E))\).

Then there is a soft open set \((G,E)\) such that \(e_F \in (G,E) \subseteq Int_s Cl_s((F_1,E) \cap Int_s(F_2,E))\). This implies that \((G,E) \subseteq Cl_s(F_1,E) \cap Cl_s(F_2,E)\). Then \((G,E) \subseteq Int_s Cl_s(F_1,E) = (F_1,E) \) and \((G,E) \subseteq Int_s Cl_s(F_2,E) = (F_2,E)\). In conclusion \((G,E) \subseteq (F_1,E) \cap (F_2,E)\), because \(e_F \in (G,E)\) it follows that \(e_F \in (F_1,E) \cap (F_2,E)\) and we are done.

(2) It show by the following example

**Example 3.2.** Let \( X = \{h_1,h_2,h_3\}, E = \{e_1,e_2\}\) and \(\tau = \{0_E,1_E, (F_1,E),(F_2,E),(F_3,E)\}\) be a soft topological space, where \((F_1,E) = \{(e_1,\{h_1\}), (e_2,\{h_1\})\}, (F_2,E) = \{(e_1,\{h_2\}), (e_2,\{h_2\})\}, (F_3,E) = \{(e_1,\{h_1,h_2\}), (e_2,\{h_1,h_2\})\}\), then \((F_1,E),(F_2,E) \in SRO(X,E)\), but then \((F_1,E) \cup (F_2,E) \notin SRO(X,E)\).

**Remark 3.4.** A soft set \((F,E)\) in a soft topological space \((X,\tau,E)\) is soft regular open set if and only if \((F^c,E)\) is soft regular closed set.

**Corollary 3.5.** Let \((X,\tau,E)\) be a soft topological space. Let \(\{(F_i,E),i \in I\} \in SRC(X,E)\).

1. \((F_1,E) \cup (F_2,E) \in SRC(X,E)\).
2. \((F_1,E) \cap (F_2,E) \notin SRC(X,E)\).

**Proof.** (1) and (2) are obvious.

**Remark 3.6.** \(0_E\) and \(1_E\) are always soft regular open set and soft regular closed set.

**Remark 3.7.** Every soft regular open set is soft open set, but the converse is not true, which follows from the following example.

**Example 3.8.** The soft topological space is same in Example 3.2. Let \((F_2,E)\) is a soft open set but it is not a soft regular open set because \(Int_s(Cl_s(F_3,E)) \neq (F_3,E)\).

**Theorem 3.9.** Let \((X,\tau,E)\) be a soft topological space. Then

1. The closure of a soft open set is a soft regular closed set.
2. The interior of a soft closed set is a soft regular open set.

**Proof.** (1) Let \((F,E)\) be a soft open set of a soft topological space \((X,\tau,E)\). Clearly, \(Int_s(Cl_s(F,E)) \subseteq (F,E)\), implies that \(Cl_s(Int_s(Cl_s(F,E))) \subseteq Cl_s(F,E)\). Now, the fact that \((F,E)\) is a soft open set implies that \(F \subseteq Int_s(Cl_s(F,E))\), and \(Cl_s(F,E) \subseteq Cl_s(Int_s(Cl_s(F,E)))\).

Thus \(Cl_s(F,E)\) is a soft regular closed set.

(2) Let \((F,E)\) be a soft closed set of a soft topological space \((X,\tau,E)\). Clearly, \(Int_s(F,E) \subseteq Cl_s(Int_s(F,E))\).

implies that \(Int_s(F,E) \subseteq Int_s Cl_s(Int_s(F,E))\). Now, the fact that \((F,E)\) is a soft closed set implies that \(Cl_s(Int_s(F,E)) \subseteq (F,E)\) and \(Int_s Cl_s(Int_s(F,E)) \subseteq Int_s(F,E)\). Thus \(Int_s(F,E)\) is a soft regular open set.

**Remark 3.10.** In a soft topological space \((X,\tau,E)\) the collection of all soft regularly open sets forms a base for some topology \(\tau_s\) on \((X,E)\).

**Definition 3.11.** In a soft topological space \((X,\tau,E)\), if \(\tau_s\) coincides with \(\tau\), then \(\tau\) is said to be a soft semi

\[\text{doi: 10.25007/ajnu.v8n4a481}\]
regularization topology.

3. Soft $\delta$-open Set And Soft $\delta$-closed Set

In this section, we define soft $\delta$–open set, soft $\delta$–closed set, soft $\delta$–interior set and, and, soft $\delta$–closure set and investigate their related properties.

**Definition 4.1.** A soft set $(U, E)$ is soft $\delta$–open set if for each $e_F \in (U, E)$, there exists a soft regular open set $(G, E)$ such that $e_F \in (G, E) \subseteq (U, E)$, denoted by $(U, E) \in S\delta O(X, E)$. A soft set $(U, E)$ is soft $\delta$–open set if it is the union of soft regular open sets. The complement of soft $\delta$–open set is said to be soft $\delta$–closed set, denoted by $(U^c, E) \in S\delta C(X, E)$.

**Proposition 4.2.** The family $\tau_\delta$ of all soft $\delta$–open sets defines a soft topology on $X$. Soft $\delta$–topology on $X$ if the following axioms are true.

1. $0_E, 1_E \in \tau_\delta$.
2. If $(U_1, E), (U_2, E) \in \tau_\delta$, then $(U_1, E) \cap (U_2, E) \in \tau_\delta$.
3. If $(G_i, E) \in \tau_\delta$ for every $i \in I$, then $\bigcup (G_i, E): i \in I \in \tau_\delta$.

**Proof.** (1) It obvious that $0_E, 1_E \in \tau_\delta$.

(2) Let $(U_1, E), (U_2, E) \in \tau_\delta$. We prove that $(U_1, E) \cap (U_2, E) \in \tau_\delta$. Let $e_F \in (U_1, E) \cap (U_2, E)$. Then $e_F \in (U_1, E)$ and $e_F \in (U_2, E)$.

There exists $(G_1, E), (G_2, E) \in SOR(X, E)$ such that $e_F \in (G_1, E) \subseteq (U_1, E)$ and $e_F \in (G_2, E) \subseteq (U_2, E)$. Then $e_F \in (G_1, E) \cap (G_2, E) \subseteq (U_1, E) \cap (U_2, E)$. But $(G_1, E) \cap (G_2, E) \in SRO(X, E)$.

Then $(U_1, E) \cap (U_2, E) \in \tau_\delta$.

(3) Let $(F_i, E), i \in I \in \tau_\delta$. We prove that $\bigcup (F_i, E), i \in I \in \tau_\delta$. Let $e_F \in \bigcup (F_i, E), i \in I$. Then $e_F \in \{ (F_i, E), i \in I \}$ and there exists $(G, E) \in SOR(X, E)$ such that $e_F \in (G, E) \subseteq \{ (F_i, E), i \in I \}$. Since $(F_i, E), i \in I \}$ $\subseteq \{ (F_i, E), i \in I \}$, then $e_F \in (G, E) \subseteq \{ (F_i, E), i \in I \}$. Thus $\{ (F_i, E), i \in I \} \in \tau_\delta$.

**Proposition 4.3.** A soft regularly open set is soft $\delta$–open set in a soft topological space $(X, \tau_\delta, E)$.

**Proof.** The proof follows from the definitions.

**Remark 4.4.** A soft $\delta$–open set need not be soft regular open set in a soft topological space $(X, \tau, E)$.

**Example 4.5.** The soft topological space is same in Example 3.2. We get $\tau_\delta = \{ 0_E, 1_E, (F_1, E), (F_2, E), (F_3, E) \}$ be a soft $\delta$–topological space. We have $(F_3, E)$ is a soft $\delta$–open set but it is not a soft regular open set because $Int_\delta(C_\delta(F_3, E)) \neq (F_3, E)$.

**Proposition 4.6.** The following statement is true.

1. $\tau_\delta \subseteq \tau$
2. $\tau \supseteq (\tau_\delta) \supseteq (\tau_\delta) \supseteq \cdots$

**Proof.** (1) Let $(F, E) \in \tau_\delta$. There exists $(G, E) \in SRO(X, E)$ such that $e_F \in (G, E) \subseteq (F, E)$; But $(G, E)$ is a soft open set. We get $(F, E)$ is neighborhood of $e_F$. Thus $(F, E) \in \tau$.

(2) Follows by part (1) and Proposition 4.2.

**Remark 4.7.** Soft regularly open set $\Rightarrow$ soft $\delta$–open set $\Rightarrow$ soft open set, but the converse in no case is true.

**Remark 4.8.** It can be observed that for any soft topological space $(X, \tau, E)$, the topologies $\tau, \tau_\delta$ are different and moreover $\tau_\delta \subseteq \tau$. It is clear that in a semi regularization space the above three topologies coincide, since $\tau_\delta = \tau$ in that case.

**Proposition 4.9.** Let $(X, \tau, E)$ be a soft topological space. The family $\tau_\delta^c$, has the following properties.

1. $0_E, 1_E \in \tau_\delta^c$.
2. If $(G, E), (H, E) \in \tau_\delta^c$, then $(G, E) \cup (H, E) \in \tau_\delta^c$.
3. If $(G_i, E) \in \tau_\delta^c$, then $(G_i, E) \in \tau_\delta^c$.

**Proof.** The proof verify directly from the proposition 4.2: and (propositions 2.10 and 2.12 of Georgiou & Megaritis, 2014).

**Definition 4.10.** A soft set $(G, E)$ in a soft $\delta$–topological space $(X, \tau_\delta, E)$, is called a soft $\delta$–neighborhood (brie: $\delta$–snbd) of a soft point $e_F \in SP(X)$ if there exists a soft $\delta$–open set $(H, E)$ such that $e_F \in (H, E) \subseteq (G, E)$.

The soft $\delta$–neighborhood system of a soft point $e_F$, denoted by $N_{\epsilon}(e_F)$, is the family of all of its soft $\delta$–neighborhoods.
Proposition 4.11. Let \((X, \tau_\delta, E)\) be soft topological space, \((U, E) \in SS(X)_E\). Then \((U, E)\) is soft \(\delta\)–open set if only if \((U, E) \in N_{\tau_\delta}(e_F)\) for every \(e_F \in (U, E)\).

Proof. Let \((U, E) \in S\delta\theta(X, E)\) and \(e_F \in (U, E)\). Thus \(e_F \in (U, E)\) \(\subseteq (U, E)\). And then \((U, E) \in N_{\tau_\delta}(e_F)\) for every \(e_F \in (U, E)\).
Conversely let \((U, E) \in N_{\tau_\delta}(e_F)\) for every \(e_F \in (U, E)\), there exists a soft \(\delta\)–open set \((H, E)_{e_F}\) such that \(e_F \in (H, E)_{e_F} \subseteq (U, E)\). Therefore \((U, E) = \cup (H, E)_{e_F} : e_F \in (U, E), (H, E)_{e_F} \in \tau_\delta\). Then \((U, E) \in S\delta\theta(X, E)\)

Proposition 4.12. Let \((X, \tau_\delta, E)\) be soft topological space, \((U, E), (G, E) \in SS(X, E)\). Then

1) If \((G, E) \in N_{\tau_\delta}(e_F)\). Then \(e_F \in (G, E)\).

2) If \((G, E), (U, E) \in N_{\tau_\delta}(e_F)\) then \((G, E) \cap (U, E) \in N_{\tau_\delta}(e_F)\).

3) If \((G, E) \in N_{\tau_\delta}(e_F)\) and \((G, E) \subseteq (U, E)\) then \((U, E) \in N_{\tau_\delta}(e_F)\).

Proof. (1) Obvious.

(2) Pick \((G, E), (U, E) \in N_{\tau_\delta}(e_F)\). There exists \((H_1, E), (H_2, E) \in S\delta\theta(X, E)\), such that \(e_F \in (H_1, E) \subseteq (G, E)\), and \(e_F \in (H_1, E) \subseteq (U, E)\). Then \(e_F \in (H_1, E) \cap (H_2, E) \subseteq (G, E) \cap (U, E)\). And then \((G, E) \cap (U, E) \in N_{\tau_\delta}(e_F)\).

(3) Let \((G, E) \in N_{\tau_\delta}(e_F), (G, E) \subseteq (U, E)\). There exists \((H, E) \in S\delta\theta(X, E)\), such that \(e_F \in (H, E) \subseteq (G, E)\). Hence \(e_F \in (H, E) \subseteq (G, E) \subseteq (U, E)\). Then \((U, E) \in N_{\tau_\delta}(e_F)\).

Definition 4.13. Let \((G, E)\) be a soft set of topological space \((X, \tau_\delta, E)\). A soft point \(e_F\) is called soft \(\delta\)–interior point of \((G, E)\) if there exists a soft \(\delta\)–open set \((U, E)\) such that \(e_F \in (U, E) \subseteq (G, E)\). The set of all soft \(\delta\)–interior points of \((G, E)\) is called the soft \(\delta\)–interior of \((G, E)\) and is denoted by \(Int^\delta\mathcal{S}(G, E)\).

In other words a soft point \(e_F\) is called its soft \(\delta\)–interior point if \((G, E)\) is the \(\delta\)–snbd of the soft point \(e_F\). So mean that \(Int^\delta\mathcal{S}(G, E) \in \tau_\delta\).

Proposition 4.14. Let \((X, \tau_\delta, E)\)be a soft topological space and \((H, E), (G, E), (U, E) \in SS(X, E)\). Then the following statements are true.

1) \(Int^\delta\mathcal{S}(U, E) \subseteq (U, E)\).

2) \(Int^\delta\mathcal{S}(U, E) = \cup \{(H, E) \in \tau_\delta, (H, E) \subseteq (U, E)\}\).

3) \(Int^\delta\mathcal{S}(U, E)\) is the largest soft \(\delta\)–open set contained in \((U, E)\).

4) \((U, E)\) is soft \(\delta\)–open set if and only if \(Int^\delta\mathcal{S}(U, E) = (U, E)\).

5) \(Int^\delta\mathcal{S}(0_E) = 0_E\) and \(Int^\delta\mathcal{S}(1_E) = 1_E\).

6) \(Int^\delta\mathcal{S}(Int^\delta\mathcal{S}(U, E)) = Int^\delta\mathcal{S}(U, E)\)

7) If \((H, E) \subseteq (G, E)\), then \(Int^\delta\mathcal{S}(H, E) \subseteq Int^\delta\mathcal{S}(G, E)\).

8) \(Int^\delta\mathcal{S}(G, E) \cap Int^\delta\mathcal{S}(G, E) \subseteq Int^\delta\mathcal{S}((H, E) \cup (G, E))\).

9) \(Int^\delta\mathcal{S}(H, E) \cap Int^\delta\mathcal{S}(G, E) \subseteq Int^\delta\mathcal{S}((H, E) \cap (G, E))\).

Proof. (1) Obvious.

(2) If \(e_F \in \cup \{\{H, E\}, \in \tau_\delta, (H, E) \subseteq (U, E)\}\). Then \(e_F \in (U, E) \subseteq (U, E)\), showing that \(e_F\) is a soft \(\delta\)–interior point of \((U, E)\) and so \(e_F \in Int^\delta\mathcal{S}(U, E)\). Thus \(\cup \{\{H, E\}, \in \tau_\delta, (H, E) \subseteq (U, E)\} \subseteq Int^\delta\mathcal{S}(U, E)\).

Conversely, let \(e_F \in Int^\delta\mathcal{S}(U, E)\), then \(e_F\) is a soft \(\delta\)–interior point of \((U, E)\), there exists \((H, E) \in \tau_\delta\) such that \(e_F \in (H, E) \subseteq (U, E)\). Consequently \(e_F \in \cup \{\{H, E\}, \in \tau_\delta, (H, E) \subseteq (U, E)\}\). Therefore \(Int^\delta\mathcal{S}(U, E) \subseteq \cup \{\{H, E\}, \in \tau_\delta, (H, E) \subseteq (U, E)\}\). We are done

(3) By (2) above \(Int^\delta\mathcal{S}(U, E) = \cup \{\{H, E\}, \in \tau_\delta, (H, E) \subseteq (U, E)\}\). Thus \(Int^\delta\mathcal{S}(U, E)\) is a soft \(\delta\)–open set of \((U, E)\).

\(Int^\delta\mathcal{S}(U, E)\) is the largest soft \(\delta\)–open set contained in \((U, E)\).

(4) Let \((U, E)\) is soft \(\delta\)–open set. Then a soft \(\delta\)–open set containing all of its soft points, it follows that every soft point of \((U, E)\) is a soft \(\delta\)–interior of \((U, E)\). Thus \(Int^\delta\mathcal{S}(U, E) \subseteq (U, E)\) Let \(e_F \in Int^\delta\mathcal{S}(U, E)\) there exists \((H, E) \in \tau_\delta\) such that \(e_F \in (H, E) \subseteq (U, E)\) then \(e_F \in (U, E)\).
\[ \text{Int}_\delta^g (U, E) \subseteq (U, E) \quad \text{Therefore} \quad \text{Int}_\delta^g (U, E) = (U, E) \]

Conversely, since \( \text{Int}_\delta^g (U, E) = (U, E) \), then \( (U, E) \in \tau_\delta \).
Hence \( (U, E) \) is soft \( \delta \) –open set if and only if \( \text{Int}_\delta^g (U, E) = (U, E) \).

(5) Is obvious.

(6) Since \( \text{Int}_\delta^g (U, E) \in S\delta O(X, E) \) is soft \( \delta \) –open set. We have \( \text{Int}_\delta^g \text{Int}_\delta^g (U, E) = \text{Int}_\delta^g (U, E) \)

(7) Let \( e_F \in \text{Int}_\delta^g (H, E) \) there exists \( (U, E) \in \tau_\delta \) soft containing \( e_F \) such that 
\( (U, E) \subseteq (H, E) \). But \( (H, E) \subseteq (G, E) \). Then \( (U, E) \subseteq (G, E) \) which implies that
\( e_F \in \text{Int}_\delta^g (G, E) \). Thus \( \text{Int}_\delta^g (H, E) \subseteq \text{Int}_\delta^g (G, E) \)

(8) Since \( (H, E) \subseteq (H, E) \cup (G, E) \). We have \( \text{Int}_\delta^g (H, E) \subseteq \text{Int}_\delta^g (H, E) \cup \text{Int}_\delta^g (G, E) \) and \( (H, E) \subseteq (H, E) \cup (G, E) \). Then
\[ \text{Int}_\delta^g (G, E) \subseteq \text{Int}_\delta^g ((H, E) \cup (G, E)) \]
Therefore
\[ \text{Int}_\delta^g ((H, E) \cup (G, E)) = \text{Int}_\delta^g (H, E) \cup \text{Int}_\delta^g (G, E). \]

(9) Since \( (H, E) \cap (G, E) \subseteq (H, E) \). Then \( \text{Int}_\delta^g ((H, E) \cap (G, E)) \subseteq \text{Int}_\delta^g (H, E) \cap \text{Int}_\delta^g (G, E). \)

Conversely, let \( e_F \in \text{Int}_\delta^g (H, E) \cap \text{Int}_\delta^g (G, E) \). So that \( e_F \in \text{Int}_\delta^g (H, E) \) and \( e_F \in \text{Int}_\delta^g (G, E) \). There exists two soft \( \delta \) –open set \( (U_1, E), (U_2, E) \) soft containing \( e_F \) such that 
\( (U_1, E) \subseteq (H, E) \), and \( (U_2, E) \subseteq (G, E) \). Implies that 
\( (U_1, E) \cap (U_2, E) = (U, E) \in S\delta O(X, E) \), and \( e_F \in (U, E) \subseteq (H, E) \cap (G, E) \). Then \( e_F \in \text{Int}_\delta^g ((H, E) \cap (G, E)) \) 
\[ \text{Int}_\delta^g ((H, E) \cap (G, E)) \subseteq \text{Int}_\delta^g (H, E) \cap \text{Int}_\delta^g (G, E) \]
Therefore \( \text{Int}_\delta^g (H, E) \cap \text{Int}_\delta^g (G, E) = \text{Int}_\delta^g ((H, E) \cap (G, E)) \). The following example shows that the equalities do not hold in Proposition 4.14(8).

Example 4.15. The soft topological space \((X, \tau), (E, \tau)\) is the same as in Example 3.2:

Suppose that \( (U, E) = \{(e_1, \{h_1\}), (e_2, \{h_2, h_3\}) \} \) and 
\( (G, E) = \{(e_1, \{h_2, h_3\}), (e_2, \{h_1, h_2\}) \} \). One can deduce that
\[ \text{Int}_\delta^g (H, E) \cup \text{Int}_\delta^g (G, E) = \text{Int}_\delta^g ((H, E) \cup (G, E)) \]
and \( \text{Int}_\delta^g (H, E) \cap \text{Int}_\delta^g (G, E) \neq \text{Int}_\delta^g ((H, E) \cap (G, E)) \).

Corollary 4.16. Let \((X, \tau_\delta, E)\) be a soft topological space, and \( (U, E) \in SS(X, E) \). Then
\[ \text{Int}_\delta^g (U, E) \subseteq \{(H_i, E) : (H_i, E) \subseteq (U, E), (H_i, E) \in SRC(X, E) \}. \]

Definition 4.17. Let \((X, \tau), (E, \tau)\) be a soft topological space and \( e_F \in S\delta P(X) \) is said to be a soft \( \delta \) –cluster point of \( (F, E) \in SS(X, E) \) if for every a soft regular open \( (U, E) \) soft containing of \( e_F \) we have \( (F, E) \cap (U, E) \neq 0 \).

The set of all soft \( \delta \) –cluster points of \( (G, E) \) is called the soft \( \delta \) –closure denoted by \( Cl_\delta^g (G, E) \)

Remark 4.18. Let \((X, \tau), (E, \tau)\) be a soft topological space and \( (F, E) \in (X, E) \). Then
\[ Cl_\delta^g (F, E) = \cap \{(H_i, E) : (F, E) \subseteq (H_i, E), (H_i, E) \in SRC(X, E) \}. \]

Proposition 4.19. Let \((X, \tau), (E, \tau)\) be a soft topological space over \( X \) and \((V, E), (F, E) \in SS(X, E) \). Then
1. \( Cl_\delta^g (V, E) \subseteq S\delta C(X, E) \).
2. \( (V, E) \subseteq Cl_\delta^g (V, E) \).
3. \( (V, E) \) is soft \( \delta \) –closed set if and only if \( Cl_\delta^g (V, E) = (V, E) \).
4. \( Cl_\delta^g (F, E) = \cap \{(H_i, E) \in S\delta C(X, E), (F, E) \subseteq (H_i, E) \}. \)
5. \( Cl_\delta^g (0_E) = 0_E \) and \( Cl_\delta^g (1_E) = 1_E \).
6. \( Cl_\delta^g (V, E) \subseteq Cl_\delta^g (V, E) \).
7. \( Cl_\delta^g Cl_\delta^g (F, E) = Cl_\delta^g (F, E) \).
8. If \( (V, E) \subseteq (F, E) \), then \( Cl_\delta^g (V, E) \subseteq Cl_\delta^g (F, E) \).
9. \( Cl_\delta^g (V, E) \cup Cl_\delta^g (G, E) = Cl_\delta^g ((V, E) \cup (G, E)) \).
10. \( Cl_\delta^g ((H, E) \cap (V, E)) \subseteq Cl_\delta^g (H, E) \cap Cl_\delta^g (V, E) \).

The following example shows that the equalities do not hold in Proposition 4.19(9).

Example 4.20. The soft topological space \((X, \tau), (E, \tau)\) is the same as in Example 3.2. Suppose \( (H, E) = \{(e_1, \{h_2\}), (e_2, \{h_2\}) \} \) and 
\( (M, E) = \{(e_1, \{h_1\}), (e_2, \{h_2, h_3\}) \} \)
\[ Cl_\delta^g (H, E) = \{(e_1, \{h_2, h_3\}), (e_2, \{h_2, h_3\}) \}, \quad Cl_\delta^g (M, E) = \{(e_1, \{h_1, h_3\}), (e_2, \{h_1, h_3\}) \}, \quad Cl_\delta^g ((H, E) \cap (M, E)) = Cl_\delta^g (0_E) = 0_E \]. Therefore \( Cl_\delta^g (H, E) \cap Cl_\delta^g (M, E) \not\subseteq Cl_\delta^g ((H, E) \cap (M, E)) \).
Proposition 4.21. Let \((X, \tau, E)\) be a soft topological space, and \((G; E) \in SS(X, E)\). Then.

1. \(Cl^\delta(E^c, E) = (Int^\delta(E, E))^c\)
2. \((Cl^\delta(V, E))^c = Int^\delta(V^c, E)\).
3. \(Int^\delta(V, E) = (Cl^\delta(V^c, E))^c\)

Proof. (1) We prove that \( Cl^\delta(E^c, E) \subseteq (Int^\delta(E, E))^c \)
Let \(e_F \in Cl^\delta(V^c, E)\). Then, for every soft regular open set \((U, E)\) containing of \(e_F\), we have \((V^c, E) \cap (U, E) \neq 0_E\).
Then the relation \((U, E) \subseteq (V, E)\) is not true.
Therefore \(e_F \notin Int^\delta(V, E)\). Then \(e_F \in (Int^\delta(V, E))^c\) thus
\(Cl^\delta(V^c, E) \subseteq (Int^\delta(V, E))^c\)
Conversely, we prove that \(Int^\delta(V, E))^c \subseteq Cl^\delta(V^c, E)\). Let \(e_F \in (Int^\delta(V, E))^c\) and \(e_F \notin Int^\delta(V, E)\). There \((U, E)\) be a soft regular open set containing of \(e_F\) such that \(e_F \subseteq (U, E) \subseteq (V^c, E)\) therefore have \((V^c, E) \cap (U, E) \neq 0_E\).
Thus, \(e_F \in Cl^\delta(V^c, E)\).
We gets \(Int^\delta(V, E))^c \subseteq Cl^\delta(V^c, E)\). Therefore \(Int^\delta(V^c, E) = (Int^\delta(V, E))^c\) .

(2) \((Cl^\delta(V, E))^c = (\cap \{ (H_i, E) \in S\delta C(X, E), (H_i, E) \subseteq (V, E) \})^c = \cup \{ (H_i^c, E) \in S\delta O(X, E), (V^c, E) \subseteq (H_i^c, E) \}\)
(3) Obvious.

3 Soft \(\delta\) –Boundary and Soft \(\delta\) –Exterior

Definition 5.1. Let \((X, \tau, E)\) be a soft topological space and \((F, E)\) be a soft set
over \(X\). The soft \(\delta\) –boundary of soft set \((F, E)\) over \(X\) is denoted by \(Bd^\delta(F, E)\) and is defined as \(Bd^\delta(F, E) = Cl^\delta(F, E) \cap Cl^\delta(F^c, E)\).

Remark 5.2. From the above definition it follows directly that the soft sets \((F, E)\)
and \((F^c, E)\) have same soft \(\delta\) –boundary.

Proposition 5.3. Let \((X, \tau, E)\) be a soft topological space, \((F, E) \in SS(X, E)\). Then the following statements are true.

1. \(Bd^\delta(F, E) = Cl^\delta(F, E) \setminus Int^\delta(F, E)\).
2. \(Bd^\delta(F, E) \cap Int^\delta(F, E) = 0_E\).
3. \((F, E) \cup Bd^\delta(F, E) = Cl^\delta(F, E)\)

Proof. (1) \(Bd^\delta(F, E) \subseteq Cl^\delta(F, E) \setminus Int^\delta(F, E)\).
(2) \(Bd^\delta(F, E) \cap Int^\delta(F, E) = 0_E\).
Conversely, suppose that \((F, E) \cap Bd^\delta(F, E) = 0_E\)
Theorem 5.8. Let \((X, \tau, E)\) be a soft topological space, \((F, E) \in SS(X, E)\). Then the following statements are true.

1. \(\text{Ext}^E_\delta(F, E) \cap \text{Int}^E_\delta(F, E) = \text{Ext}^E_\delta(F, E) \cap \text{Int}^E_\delta(F, E) = 0_E\) \(\text{Int}^E_\delta(F, E) \cap \text{Int}^E_\delta(F, E) = 0_E\)

2. If \((F, E)\) is soft \(\delta\)–closed set, then \((F, E) \cap \text{Int}^E_\delta(F, E) = \text{Int}^E_\delta(F, E) \cap \text{Int}^E_\delta(F, E) = 0_E\).

\textbf{Proof.} (1) We have \((F, E) \cap \text{Int}^E_\delta(F, E) = \text{Int}^E_\delta(F, E) \cap \text{Int}^E_\delta(F, E) \cap \text{Int}^E_\delta(F, E) = 0_E\),

\((F, E) \cap \text{Int}^E_\delta(F, E) \cap \text{Int}^E_\delta(F, E) = 0_E\).

(2) Since \((F, E)\) is soft \(\delta\)–closed set, \(\text{Cl}^E_\delta(F, E) = (F, E)\).

Therefore \((F, E) \cap \text{Int}^E_\delta(F, E) = \text{Cl}^E_\delta(F, E) \cap \text{Int}^E_\delta(F, E) = \text{Int}^E_\delta(F, E) \cap \text{Int}^E_\delta(F, E) = 0_E\).

Definition 5.7. Let \((X, \tau, E)\) be a soft topological space and \((F, E) \in SS(X, E)\). The \(\delta\)–exterior of \((F, A)\) is the soft set. \(\text{Ext}^E_\delta(F, E) = \text{Int}^E_\delta(F, E)\) is said to be a soft \(\delta\)–exterior of \((F, A)\).

Theorem 5.8. Let \((X, \tau, E)\) be a soft topological space and \((F, E), (G, E) \in SS(X, E)\). Then the following statements hold.

1. \(\text{Ext}^E_\delta(F, E) \subseteq \text{Ext}(F, E)\) is soft \(\delta\)–open set.

2. \(\text{Ext}^E_\delta(F, E)\) is soft \(\delta\)–open set.
In Example 3.2 (3), we have $\text{Int}_S^\delta (F,E) \subseteq \text{Ext}_S^\delta \left( \text{Ext}_S^\delta (F,E) \right)$.

4 Soft $f_{pu} - \delta$ - continuity

In this section, we introduce the notion of soft $f_{pu} - \delta$ - continuity of functions induced by two mappings $u : X \to Y$ and $p : E \to B$ on soft topological spaces $(X, \tau, E)$ and $(Y, \tau^*, B)$.

Definition 6.1. Let $(X, \tau, E)$ and $(Y, \tau^*, B)$ be two soft topological spaces, and $e_F \in SP(X)$. The map $f_{pu} : SS(X,E) \to SS(Y,B)$ is soft $f_{pu} - \delta$ - continuous at $e_F$ if for each $(G,B)$ neighborhood $e_F$, there exists a $(H,E)$ neighborhood $e_F$ such that $f_{pu}(\text{Int}_S(CI_S(H,E))) \subseteq (\text{Int}_S(CI_S(G,B)))$.

Theorem 6.2. For the mapping $f_{pu} : SS(X,E) \to SS(Y,B)$, the following properties are equivalent.

1. $f_{pu}$ is soft $\delta$ - continuous.
2. For each $e_F \in SP(X)$, and each soft regular open set $(V,B)$ containing $f_{pu}(e_F)$, there exists a soft regular open set $(U,E)$ containing $e_F$ such that $f_{pu}(U,E) \subseteq (V,B)$.
3. $f_{pu}CI_S^\delta(A,E) \subseteq CI_S^\delta f_{pu}(A,E)$ for every $(A,E) \in SS(X,E)$.
4. $CI_S^\delta f_{pu}^{-1}(N,B) \subseteq f_{pu}^{-1}CI_S^\delta(N,B)$ for every $(N,B) \in SS(Y,B)$.
5. For each soft $\delta$ - closed set $(V,B) \in S\delta C(Y,B)$, $f_{pu}^{-1}(V,B) \in S\delta C(X,E)$.
6. For each soft $\delta$ - open set $(V,B) \in S\delta O(Y,B)$, $f_{pu}^{-1}(V,B) \in S\delta O(X,E)$.
7. For each soft regular open set $(V,B) \in SS(Y,B)$, $f_{pu}^{-1}(V,B) \in S\delta O(X,E)$.
8. For each soft regular closed set $(N,B) \in SS(Y,B)$, $f_{pu}^{-1}(N,B) \in S\delta C(X,E)$.

Proof. (1) $\Rightarrow$ (2): Directly from Definition 6.1.

(2) $\Rightarrow$ (3): Let $e_F \in SP(X)$ and $(A,E) \in SS(X,E)$ such that $f_{pu}(e_F) \in f_{pu}CI_S^\delta(A,E)$. Suppose that $f_{pu}(e_F) \neq f_{pu}CI_S^\delta(A,E)$. Then, there exists a soft regular open set neighborhood $(V,B)$ of $f_{pu}(e_F)$ such that $f_{pu}(A,E) \cap (V,B) = 0_B$. By (2), there exists a soft regular open set neighborhood $(U,E)$ of $(e_F)$ such that $f_{pu}(U,E) \subseteq (V,B)$. Since $f_{pu}(A,E) \cap f_{pu}(U,E) \subseteq f_{pu}(A,E) \cap (V,B) = 0_B$, $f_{pu}(A,E) \subseteq f_{pu}(U,E)$. Hence, we get that $(A,E) \cap (U,E) \subseteq f_{pu}^{-1}(f_{pu}(A,E)) \cap f_{pu}^{-1}(f_{pu}(U,E)) = \mu^{-1}\left(f_{pu}(A,E) \cap f_{pu}(U,E)\right) = 0_B$. Hence we have $(A,E) \subseteq 0_B$, and $(e_F) \notin CI_S^\delta(A,E)$. This shows that $f_{pu}(e_F) \notin f_{pu}CI_S^\delta(A,E)$. This is a contradiction.

Therefore, we obtain that $f_{pu}(e_F) \in CI_S^\delta(f_{pu}(A,E))$ (3) $\Rightarrow$ (4): Let $(N,B) \in SS(Y,B)$ such that $(A,E) = f_{pu}^{-1}(N,B)$. By (3), $f_{pu}(CI_S^\delta(f_{pu}^{-1}(N,B))) \subseteq CI_S^\delta\left(f_{pu}\left(f_{pu}^{-1}(N,B)\right)\right) \subseteq CI_S^\delta\left(f_{pu}^{-1}(N,B)\right)$. From here, we have $CI_S^\delta\left(f_{pu}^{-1}(N,B)\right) \subseteq f_{pu}^{-1}(CI_S^\delta\left(f_{pu}\left(f_{pu}^{-1}(N,B)\right)\right)) \subseteq f_{pu}^{-1}(CI_S^\delta\left(f_{pu}^{-1}(N,B)\right))$. Thus we obtain $CI_S^\delta\left(f_{pu}^{-1}(N,B)\right) \subseteq f_{pu}^{-1}(CI_S^\delta\left(f_{pu}^{-1}(N,B)\right))$. (4) $\Rightarrow$ (5): Let $(V,B) \in S\delta C(Y,B)$. By (4), $CI_S^\delta\left(f_{pu}^{-1}(V,B)\right) \subseteq f_{pu}^{-1}\left(CI_S^\delta\left(f_{pu}\left(f_{pu}^{-1}(V,B)\right)\right)\right) = f_{pu}^{-1}(CI_S^\delta\left(f_{pu}^{-1}(V,B)\right))$. Hence that $f_{pu}^{-1}(V,B) \in S\delta C(X,E)$.

(5) $\Rightarrow$ (6): Let $(V,B) \in S\delta O(Y,B)$. Then $(V^c,B) \in S\delta C(Y,B)$. By (5), $f_{pu}^{-1}(V^c,B) = \left(f_{pu}^{-1}(V,B)\right)^c \in S\delta C(Y,B)$ and $(V^c,B) \in S\delta O(Y,B)$. Therefore, $f_{pu}^{-1}(V^c,B) \in S\delta C(Y,B)$.

(6) $\Rightarrow$ (7): Let $(V,B) \in SRO(Y,B)$. Since every a soft regular open set is soft $\delta$ - open set, $(V,B) \in S\delta O(Y,B)$. By (6), $f_{pu}^{-1}(V,B) \in S\delta O(X,E)$.

(7) $\Rightarrow$ (8): Let $(N,B) \in SRC(Y,B)$. Then $(N^c,B) \in SRO(Y,B)$. By (7), $f_{pu}^{-1}(N^c,B) = \left(f_{pu}^{-1}(N,B)\right)^c \in S\delta O(X,E)$. Therefore, $f_{pu}^{-1}(N,B) \in S\delta C(X,E)$.

(8) $\Rightarrow$ (1): Let $e_F \in SP(X)$ and $(A,E) \in SRO(Y,B)$ such that $f_{pu}(e_F) \in (A,E)$. Now, then $(A^c,B) \in SRO(X,E)$.

doi : 10.25007/ajnu.v8n4a481
By (8), $f_{pu}^{-1}(A', B) = (f_{pu}^{-1}(A, B))^c \in S\delta C(X, E)$. Thus, $f_{pu}^{-1}(A, B) \in S\delta O(X, E)$. Since $(e_F) \in f_{pu}^{-1}(A, B)$. Then there exists $(U, E) \in SRO(Y, B)$ such that $e_F \in (U, E) \in f_{pu}^{-1}(A, B)$. Hence $f_{pu}(Int_s(Cl_u(E, U)) \subseteq (Int_s(Cl_u(A, B)))$. This shows that $f_{pu}$ is a soft $\delta$ –continuous mapping.

**Theorem 6.3.** For the mapping $f_{pu}$: $SS(X, E) \rightarrow SS(Y, B)$ , the following properties are equivalent.

1. $f_{pu}$ is soft $\delta$ –continuous.
2. $f_{pu}^{-1}Int_s^\delta(U, B) \subseteq Int_s^\delta f_{pu}^{-1}(U, B)$ for every $(U, B) \in SS(Y, B)$.

**Proof.** (1) $\Rightarrow$ (2): Let $f_{pu}$ is soft $\delta$ –continuous and $(U, B) \in SS(Y, B)$ implies that

$$Cl_s^\delta f_{pu}^{-1}(U', B) \subseteq f_{pu}^{-1}Cl_s^\delta(U', B).$$

Then

$$f_{pu}^{-1}Int_s^\delta(U, B) = f_{pu}^{-1}(Cl_s^\delta(U', B))^c = (f_{pu}^{-1}(Cl_s^\delta(U', B)))^c \subseteq (Cl_s^\delta(f_{pu}^{-1}(U', B)))^c = Cl_s^\delta(f_{pu}^{-1}(U', B))^c = Int_s^\delta f_{pu}^{-1}(U, B).$$

**6. References**


**5. Conclusion**

All over the globe, soft set theory is a topic of interest for many authors working in diverse areas due to its rich potential for applications in several directions. So, we found it reasonable to extend some known concept in general topology to the soft topological structures. In this paper, several characterizations of soft $\delta$ –topology in terms of soft $\delta$ –open sets are introduced and the concept of soft $f_{pu} - \delta$ –continuity is obtained. Thus we fill a gap in the existing literature on soft topology.