Some Properties of Soft Delta-Topology

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ABSTRACT

In this paper, we apply the concept of soft sets to δ-open set and δ-closed set. The associated soft δ-topology in terms of soft δ-open sets were introduced and some properties of them were investigated. Moreover, the definitions, characterizations and basic results concerning soft δ-interior, soft δ-closure, soft δ-boundary and soft δ-exterior were given. Finally, the concept of soft pu−δ- continuity was defined and some properties of it were introduced.

Keywords: Soft set; soft δ-open set; soft δ-topology; soft pu−δ-continuity.

1. Introduction

Some concepts in mathematics can be considered as mathematical tools for dealing with uncertainties, namely theory of vague sets, theory of rough sets and etc. But all of these theories have their own difficulties. The concept of soft sets was first introduced by Molodtsov as a general mathematical tool for dealing with uncertain objects (Dmitriy Molodtsov, 1999). He successfully applied the soft theory in several directions, such as smoothness of functions, game theory, probability, Perron integration, Riemann integration, theory of measurement(Dmitriy Molodtsov, 1999; DA Molodtsov, 2001; D Molodtsov, 2004; D Molodtsov, Leonov, & Kovkov, 2006). It is remarkable that, Molodtsov used this concept in order to solve complicated problems in other sciences such as, engineering, economics and etc. The soft set theory has been applied to many different fields. Later, few researches((Aygünoğlu & Aygün, 2012),(Çağman, Karataş, & Enginoglu, 2011),(Georgiou & Megaritis, 2014),(Hussain & Ahmad, 2011),(Min, 2011),(Peyghan, Samadi, & Tayebi),(Shabir & Naz, 2011),(Zorlutuna, Akdag, Min, & Atmaca, 2012)) introduced and studied the notion of soft topological spaces. Recently, in (2014), S. Yüksel, N. Tozlu and Z. G. Ergül (Ergül, Yüksel, & Tozlu, 2014) initiated soft regular open set and soft regular closed set.

2. Preliminaries

We present here the basic definitions and results related to soft set theory that will be needed in the sequel.

Definition 2.1.(Dmitriy Molodtsov, 1999) Let X to be as an initial universe and E as set of parameters, and $P(X)$ be denoted as the power set of X. A pair (F, A) is referred to as a soft set over X, where F is a mapping given by $F: A \to P(X)$ where and $A \sqsubset E$. On the other hand, a soft set over X is a parameterized family of subsets of the universe X, for $\varepsilon \in A$, $F(\varepsilon)$ might be recognized as the set of ε −approximate elements of the soft set (F, A). The set of all these soft sets over X denoted by $SS(X)_A$.

Definition 2.2.(Maji, Biswas, & Roy, 2003). Let$(F, A), (G, B) \in SS(X)_E$. Then (F, A) is a soft subset of(G, B), denoted by$(F, A) \sqsubseteq (G, B)$, if (i) $A \sqsubseteq B$, (ii) $F(e) \sqsubseteq G(e)$, $\forall e \in A$. In this case, (F, A) is said to be a soft subset of (G, B) and (G, B) is said to be a soft superset of(F, A).

Definition 2.3.(Maji et al., 2003). Two soft subsets (F, A)

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and (G, B) over a common universe set X are said to be soft equal if (F, A) is a soft subset of (G, B) and (G, B) is a soft subset of (F, A).

Definition 2.4.(Ali, Feng, Liu, Min, & Shabir, 2009). The complement of a soft set (F, A), denoted by $(F, A)^c$, is defined by $(F, A)^c = (F^c, A)$, $F^c: A \to P(X)$ is a functions given by $F^c(e) = X - F(e)$, $\forall e \in A$. Clearly $((F, A)^c)^c$ is the same as (F, A).

Definition 2.5. (Maji et al., 2003). A soft set (F, E) over X is said to be a null soft set, denoted by 0_E if $F(e) = \emptyset, \forall e \in A$, A soft set (F, E) over X is said to be an absolute soft set, denoted by 1_E if $F(e) = X, \forall e \in E$

Definition 2.6. (Zorlutuna et al., 2012). The soft set $(F, A) \in SS(X)_E$ is called a soft point in X, denoted by e_F, if for the element $e \in E, F(e) \neq 0_E$ and $F(e') = 0_E \forall e' \in E \backslash \{e\}$. The set of all soft points of X is denoted by $SP(X)$. The soft point e_F is said to be in the soft set (G, E), denoted by $e_F \in (G, E)$, if for the element $e_F \in E, F(e) \sqsubseteq G(e)$.

Definition 2.7. (Peyghan et al.; Shabir & Naz, 2011). Let X be an initial universe set, E a set of parameters and $\tau \sqsubseteq SS(X)_E$. We say that the family τ a soft topology on X if the following axioms are true.

(1) $0_E , 1_E \in \tau$.

(2) If $(G, E), (H, E) \in \tau$, then $(G, E) \sqcap (H, E) \in \tau$.

(3) If $(Gi, E) \in \tau$ for every $i \in I$, then $\sqcup (Gi, E): i \in I\} \in \tau$. The triplet (X, τ, E) is called a soft topological space. The members of τ are called soft open sets in X. Also, a soft set (F, E) is called soft closed set if the complement (F^c, E) belongs to τ. The family of all soft closed sets is denoted by τ^c

Definition 2.8. (Zorlutuna et al., 2012). Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. A soft topology $\tau_{(F,E)} = \{(G, E) \sqcap (F, E) : (G, E) \in \tau\}$ is called a soft relative topology of τ on (F, E), and $((F, E), \tau_{(F,E)})$ is called a soft subspace of (X, τ, E).

Definition 2.9. (Zorlutuna et al., 2012). A soft set (G, E) in a soft topological space $(X, \tau, E)$is called

(i) a soft neighborhoods of a soft point $F(e) \in (X, \tau, E)$if there exists a soft open set (H, E) such that $F(e) \in (H, E) \sqsubseteq (G, E)$.

(ii) a soft neighborhood of a soft set (F, E) if there exists a soft open set (H, E) such that$(F, E) \sqsubseteq (H, E) \sqsubseteq (G, E)$. The neighbourhood system of a soft point $F(e)$ denoted by $N_\tau(F(e))$, is the family of all its neighborhood.

Definition 2.11. (Peyghan et al.; Shabir & Naz, 2011), (Zorlutuna et al., 2012) Let (X, τ, E) be a soft topological space and $(F, E) \in SS(X)_E$. The soft closure of (F, E) [24] is the soft set $Cl_s(F, E) = \sqcap \{(S, E): (S, E) \in \tau^c, (F, E) \sqsubseteq (S, E)\}$.

The soft interior of (F, E) [29] is the soft set $Int_s(F, E) = \sqcup \{(S, E): (S, E) \in \tau, (S, E) \sqsubseteq (F, E)\}$.

Theorem 3.12. (Zorlutuna et al., 2012). Let (X, τ, E) be a soft topological space and $(F, E), (G, E) \in SS(X)_E$. Then

(1) $(Cl_s(G, E))^c = Int_s(G^c, E)$.

(2) $(Int_s(G, E))^c = Cl_s(G^c, E)$.

Definition 2.13. (Ergül et al., 2014). Let (X, τ, E) be soft topological space and (F, E) be a soft set over X:

(1) (F, E) is said to be a soft regular open set in X if $(F, E) = Int_s(Cl_s(F; E)$, denoted by $(F, E) \in SRO(X, E)$.

(2) (F, E) is said to be a soft regular closed set in X if $(F, E) = Cl_s(Int_s(F, E)$, denoted by $(F, E) \in SRC(X, E)$).

Remark 2.14. (Ergül et al., 2014). Every soft regular open set in soft topological space Let (X, τ, E) is soft open set.

Definition 2.15.(Zorlutuna et al., 2012). Let (X, τ, E) and (Y, ρ, H) be soft topological spaces. Let $\mu: X \to Y$ and $p: E \to H$ be functions. Then the function$f_{p\mu}: SS(X)_E \longrightarrow SS(Y)_H$ is defined by:

(i) Let $(F, E) \in SS(X)_E$. The image of (F, E) under $f_{p\mu}$, written $f_{p\mu}(F, E) = ((f_{p\mu}F), p(E))$ is a soft set in $SS(Y)_H$ such that

$$f_{p\mu}(F) = \begin{cases} \sqcup_{x \in p^{-1}(y) \cap A}\, \mu(F(x)) , p^{-1}(y) \sqcap A \neq \emptyset \\ \emptyset, \text{othereise} \end{cases}$$ for all

$y \in H$.

(ii) Let $(G,H) \in SS(Y)_H$. The inverse image of (G,H) under $f_{p\mu}$, written as $f_{p\mu}^{-1}(G,H) = (f_{p\mu}^{-1}(G), p(H))$ is soft set in $SS(X)_E$ such that

$$f_{p\mu}^{-1}(G) = \begin{cases} \mu^{-1}\left(G\left(p(x)\right)\right), & p(x) \in H \\ \emptyset, & \text{othereise} \end{cases} \quad \text{for all } x \in E.$$

3. Soft regular open set and soft regular closed set

In this section, we introduce some properties of the soft regular open set and soft regular closed set.

Theorem 3.1. Let (X,τ,E) be a soft topological space. Let $\{(F_i,E), i \in I\} \in SRO(X,E)$. Then.

(1) $(F_1,E) \sqcap (F_2,E) \in SRO(X,E)$.

(2)$(F_1,E) \sqcup (F_2,E) \notin SRO(X,E)$.

Proof. (1) Let) $(F_1,E),(F_2,E) \in SRO(X,E)$. Then) $(F_1,E) \sqsubseteq Cl_s(F_1,E), (F_2,E) \sqsubseteq Cl_s(F_2,E)$, and $(F_1,E) \sqcap (F_2,E)$ is soft open set. We have $Int_s((F_1,E) \sqcap (F_2,E)) = Int_s(F_1,E) \sqcap Int_s(F_2,E)$. Then $Int_s((F_1,E) \sqcap (F_2,E)) \sqsubseteq Int_sCl_s((F_1,E) \sqcap Int_s(F_2,E))$,therefore $\left((F_1,E) \sqcap (F_2,E)\right) \sqsubseteq Int_sCl_s\left((F_1,E) \sqcap Int_s(F_2,E)\right)$.Conversely, pick $e_F \in Int_sCl_s\left((F_1,E) \sqcap Int_s(F_2,E)\right)$.

Then there is a soft open set (G,E) such that $e_F \in (G,E) \sqsubseteq Int_sCl_s\left((F_1,E) \sqcap Int_s(F_2,E)\right)$. This implies that $(G,E) \sqsubseteq Cl_s(F_1,E)$ and $(G,E) \sqsubseteq Cl_s(F_2,E)$. Then $(G,E) \sqsubseteq Int_s\, Cl_s(F_1,E) = (F_1,E)$ and $(G,E) \sqsubseteq Int_sCl_s(F_2,E) = (F_2,E)$. In conclusion $(G,E) \sqsubseteq (F_2,E) \sqcap (F_2,E)$, because $e_F \in (G,E)$ it follows that $e_F \in (F_2,E) \sqcap (F_2,E)$ and we are done.

(2) It show by the following example

Example 3.2. Let $X = \{h_1,h_2,h_3\}$, $E = \{e_1,e_2\}$ and $\tau = \{0_E, 1_E, (F_1,E),(F_2,E),(F_3,E)\}$ be a soft topological space, where $(F_1,E) = \{(e_1,\{h_1\}),(e_2,\{h_1\})\}, (F_2,E) = \{(e_1,\{h_2\}),(e_2,\{h_2\})\}, (F_3,E) = \{(e_1,\{h_1,h_2\}),(e_2,\{h_1,h_2\})\}$, then $(F_1,E),(F_2,E) \in SRO(X,E)$, but then $(F_1,E) \sqcup (F_2,E) \notin SRO(X,E)$

Remark 3.4. A soft set (F,E) in a soft topological space (X,τ,E) is soft regular open set if and only if (F^c,E) is soft regular closed set.

Corollary 3.5. Let (X,τ,E) be a soft topological space. Let $\{(F_i,E), i \in I\} \in SRC(X,E)$. Then.

(1) $(F_1,E) \sqcup (F_2,E) \in SRC(X,E)$.

(2)$(F_1,E) \sqcap (F_2,E) \notin SRC(X,E)$.

Proof. (1) and (2) are obvious.

Remark 3.6. 0_E and 1_E are always soft regular open set and soft regular closed set.

Remark 3.7. Every soft regular open set is soft open set, but the converse is not true, which follows from the following example.

Example 3.8. The soft topological space is same in Example 3.2. Let (F_3,E) is a soft open set but it is not a soft regular open set because $Int_s\left(Cl_s(F_3,E)\right) \neq (F_3,E)$.

Theorem 3.9. Let (X,τ,E) be a soft topological space. Then.

(1) The closure of a soft open set is a soft regular closed set.

(2) The interior of a soft closed set is a soft regular open set.

Proof. (1) Let (F,E) be a soft open set of a soft topological space $(X,\tau,E$. Clearly, $Int_s(Cl_s(F;E) \sqsubseteq (F,E)$, implies that $Cl_s\left(Int_s\left(Cl_s(F;E)\right)\right) \sqsubseteq Cl_s(F,E)$, . Now, the fact that (F,E) is a soft open set implies that $(F,E) \sqsubseteq Int_s(Cl_s(F;E)$, and $Cl_s(F,E) \sqsubseteq Cl_sInt_s\left(Cl_s(F;E)\right)$. Thus $Cl_s(F;E)$ is a soft regular closed set.

(2) Let (F,E) be a soft closed set of a soft topological space (X,τ,E) . Clearly, $Int_s(F,E) \sqsubseteq Cl_sInt_s(F,E)$. implies that $Int_s(F,E) \sqsubseteq Int_s\, Cl_s\, Int_s(F,E)$. Now, the fact that (F,E) is a soft closed set implies that $Cl_s\, Int_s(F,E) \sqsubseteq (F,E)$ and $Int_s\, Cl_s\, Int_s(F,E) \sqsubseteq Int_s(F,E)$. Thus $Int_s(F,E)$ is a soft regular open set.

Remark 3.10. In a soft topological space (X,τ,E) , the collection of all soft regularly open sets forms a base for some topology τ_s on (X,E).

Definition 3.11. In a soft topological space (X,τ,E), if τ_s coincides with τ , then τ is said to be a soft semi

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regularization topology.

3. Soft δ-open Set And Soft δ-closed Set

In this section, we define soft $\delta-$open set, soft $\delta-$closed set, soft $\delta-$interior set and, and, soft $\delta-$closure set and investigate their related properties.

Definition 4.1. A soft set (U, E) is soft $\delta-$open set if for each $e_F \in (U, E)$, there exists a soft regular open set (G, E) such that $e_F \in (G, E) \sqsubseteq (U, E)$, denoted by $(U, E) \in S\delta O(X, E)$ i.e. A soft set is soft $\delta-$open set if it is the union of soft regular open sets. The complement of soft $\delta-$open set is said to be soft $\delta-$closed set, denoted by $(U^C, E) \in S\delta C(X, E)$.

Proposition 4.2. The family τ_δ of all soft $\delta-$open sets defines a soft topology on X. Soft $\delta-$topology on X if the following axioms are true.

(1) $0_E, 1_E \in \tau_\delta$.

(2) If $(U_1, E), (U_2, E) \in \tau$, then $(U_1, E) \sqcap (U_2, E) \in \tau_\delta$.

(3) $If\ (Gi, E) \in \tau_\delta$ for every $i \in I$, then $\sqcup(Gi, E): i \in I\} \in \tau_\delta$.

Proof. (1) It obvious that $0_E, 1_E \in \tau_\delta$.

(2) Let $(U_1, E), (U_2, E) \in \tau_\delta$. We prove that $(U_1, E) \sqcap (U_2, E) \in \tau_\delta$. Let $e_F \in (U_1, E) \sqcap (U_2, E)$. Then $e_F \in (U_1, E)$ and $e_F \in (U_2, E)$. There exists $(G_1, E), (G_2, E) \in SOR(X, E)$ such that $e_F \in (G_1, E) \sqsubseteq (U_1, E)$ and $e_F \in (G_2, E) \sqsubseteq (U_2, E)$. Then $e_F \in (G_1, E) \sqcap (G_2, E) \sqsubseteq (U_1, E) \sqcap (U_2, E)$. But $(G_1, E) \sqcap (G_2, E) \in SRO(X, E)$.

Then $(U_1, E) \sqcap (U_2, E) \in \tau_\delta$

(3) Let Let $\{(F_i, E), i \in I\} \in \tau_\delta$. We prove that $\sqcup \{(F_i, E), i \in I\} \in \tau_\delta$. Let $e_F \in \sqcup \{(F_i, E), i \in I\}$ .Then $e_F \in \{(F_i, E), i \in I\}$ and there exists exists $(G, E) \in SOR(X, E)$ such that $e_F \in (G, E) \sqsubseteq \{(F_i, E), i \in I\}$ Since $\{(F_i, E), i \in I\} \sqsubseteq \sqcup \{(F_i, E), i \in I\}$. Then $e_F \in (G, E) \sqsubseteq \sqcup \{(F_i, E), i \in I\}$ Thus $\{(F_i, E), i \in I\} \in \tau_\delta$.

Proposition 4.3. A soft regularly open set is soft $\delta-$open set in a soft topological space (X, τ, E).

Proof. The proof follows from the definitions.

Remark 4.4. A soft $\delta-$open set need not be soft regular open set in a soft topological space (X, τ, E)

Example 4.5. The soft topological space is same in Example 3.2. We get $\tau_\delta = \{0_E, 1_E, (F_1, E), (F_2, E), (F_3, E)\}$ be a soft $\delta-$topological space. We have (F_3, E) is a soft $\delta-$open set but it is not a soft regular open set because $Int_s(Cl_s(F_3, E)) \neq (F_3, E)$.

Proposition 4.6. The following statement is true.

(1) $\tau_\delta \sqsubseteq \tau$

(2) $\tau \sqsupseteq \tau_\delta \sqsupseteq (\tau_\delta)_\delta \sqsupseteq ((\tau_\delta)_\delta)_\delta \sqsupseteq \cdots$

Proof. (1) Let $(F, E) \in \tau_\delta$ 2. There exists $(G, E) \in SRO(X, E)$ such that $e_F \in (G, E) \sqsubseteq (F, E)$; But (G, E) is a soft open set. We get (F, E) is neighborhood of e_F. Thus $(F, E) \in \tau$.

(2) Follows by part (1) and Proposition 4.2.

Remark 4.7. Soft regularly open set $\Rightarrow$ soft $\delta-$open set $\Rightarrow$ soft open set, but the converse in no case is true.

Remark 4.8. It can be observed that for any soft topological space (X, τ, E), the topologies τ, τ_s and τ_δ are different and moreover $\tau_s \sqsubseteq \tau_\delta \sqsubseteq \tau$. It is clear that in a semi regularization space the above three topologies coincide, since $\tau_s = \tau$ in that case.

Proposition 4.9. Let (X, τ, E) be a soft topological space. The family τ_δ^c, has the following properties.

(1) $0_E, 1_E \in \tau_\delta^c$.

(2) If $(G, E), (H, E) \in \tau_\delta^c$, then $(G, E) \sqcup (H, E) \in \tau_\delta^c$.

(3) $If\ (Gi, E) \in \tau_\delta^c$ for every $i \in I$, then $\sqcap (Gi, E): i \in I\} \in \tau_\delta^c$.

Proof. The proof verify directly from the proposition 4.2: and (propositions 2.10 and 2.12 of(Georgiou & Megaritis, 2014)).

Definition 4.10. A soft set (G, E) in a soft $\delta-$topological space (X, τ_δ, E), is called a soft $\delta-$neighborhood (briey: $\delta-$snbd) of a soft point $e_F \in \boldsymbol{SP(X)}$ if there exists a soft $\delta-$open set (H, E) such that $e_F \in (H, E) \sqsubseteq (G, E)$. The soft $\delta-$neighborhood system of a soft point e_F , denoted by $N_{\tau_\delta}(e_F)$, is the family of all of its soft $\delta-$neighborhoods.

Proposition 4.11. Let (X,τ_δ,E) be soft topological space , $(U,E) \in SS(X)_E$. Then (U,E) is soft δ −open set if only if $(U,E) \in N_{\tau_\delta}(e_F)$ for every $e_F \in (U,E)$.

Proof. Let $(U,E) \in S\delta O(X,E)$ and $e_F \in (U,E)$ Thus $e_F \in (U,E) \sqsubseteq (U,E)$. And then $(U,E) \in N_{\tau_\delta}(e_F)$ for every $e_F \in (U,E)$

Conversely Let $(U,E) \in N_{\tau_\delta}(e_F)$ for every $e_F \in (U,E)$, there exists a soft δ −open set $(H,E)_{e_F}$ such that $e_F \in (H,E)_{e_F} \sqsubseteq (U,E)$. Therefore $(U,E) = \sqcup (H,E)_{e_F} : e_F \in (U,E), (H,E)_{e_F} \in \tau_\delta$. Then $(U,E) \in S\delta O(X,E)$

Proposition 4.12. Let (X,τ_δ,E) be soft topological space, $(U;E),(G,E) \in SS(X,E)$. Then

(1) If $(G,E) \in N_{\tau_\delta}(e_F)$. Then $e_F \in (G,E)$.

(2) If $(G,E),(U,E) \in N_{\tau_\delta}(e_F)$ then $(G,E) \sqcap (U,E) \in N_{\tau_\delta}(e_F)$.

(3) If $(G,E) \in N_{\tau_\delta}(e_F)$ and $(G,E) \sqsubseteq (U,E)$ then $(U,E) \in N_{\tau_\delta}(e_F)$.

Proof. (1) Obvious.

(2) Pick $(G,E),(U,E) \in N_{\tau_\delta}(e_F)$. There exists $(H_1,E),(H_2,E) \in S\delta O(X,E)$, such that $e_F \in (H_1,E) \sqsubseteq (G,E)$, and $e_F \in (H_1,E) \sqsubseteq (U,E)$. Then $e_F \in (H_1,E) \sqcap (H_2,E) \sqsubseteq (G,E) \sqcap (U,E)$. Thus $(G,E) \sqcap (U,E) \in N_{\tau_\delta}(e_F)$

(3) Let $(G,E) \in N_{\tau_\delta}(e_F)$, $(G,E) \sqsubseteq (U,E)$. There exists $(H,E) \in S\delta O(X,E)$, such that $e_F \in (H,E) \sqsubseteq (G,E)$. Hence $e_F \in (H,E) \sqsubseteq (G,E) \sqsubseteq (U,E)$ .Then $(U,E) \in N_{\tau_\delta}(e_F)$.

Definition 4.13. Let (G,E) be a soft set of topological space (X,τ_δ,E). A soft point e_F is called soft δ −interior point of (G,E) if there exists a soft δ −open set (U,E) such that $e_F \in (U,E) \sqsubseteq (G,E)$. The set of all soft δ −interior points of (G,E) is called the soft δ −interior of (G,E) and is denoted by $Int_s^\delta(G,E)$.

In other words a soft point e_F is called its soft δ −interior point if (G,E) is the δ −snbd of the soft point e_F .So mean that $Int_s^\delta(G,E) \in \tau_\delta$.

Proposition 4.14. Let (X,τ_δ,E)be a soft topological space and $(H,E),(G,E),(U,E) \in SS(X,E)$. Then the following statements are true.

(1) $Int_s^\delta(U,E) \sqsubseteq (U,E)$.

(2) $Int_s^\delta(U,E) = \sqcup \{(H_i,E) \in \tau_\delta, (H_i,E) \sqsubseteq (U,E)\}$.

(3) $Int_s^\delta(U,E)$ is the largest soft δ −open set contained in (U,E).

(4) (U,E) is soft δ −open set if and only if $Int_s^\delta(U,E) = (U,E)$.

(5) $Int_s^\delta(0_E) = 0_E$ and $Int_s^\delta(1_E) = 1_E$.

(6) $Int_s^\delta(Int_s^\delta(U,E)) = Int_s^\delta(U,E)$

(7) If $(H,E) \sqsubseteq (G,E)$, then $Int_s^\delta(H,E) \sqsubseteq Int_s^\delta(G,E)$.

(8) $Int_s^\delta(H,E) \sqcup Int_s^\delta(G,E) \sqsubseteq Int_s^\delta((H,E) \sqcup (G,E))$.

(9) $Int_s^\delta(H,E) \sqcap Int_s^\delta(G,E) = Int_s^\delta((H,E) \sqcap (G,E))$.

Proof. (1) Obvious.

(2) If $e_F \in \sqcup_{i\in I} \{(H_i,E),\} \in \tau_\delta, (H_i,E) \sqsubseteq (U,E)\}$. Then $e_F \in (H_1,E) \sqsubseteq (U,E)$, showing that e_F is a soft δ −interior point of (U,E) and so $e_F \in Int_s^\delta(U,E)$. Thus $\sqcup_{i\in I} \{(H_i,E),\} \in \tau_\delta, (H_i,E) \sqsubseteq (U,E)\} \sqsubseteq Int_s^\delta(U,E)$

Conversely, let $e_F \in Int_s^\delta(U,E)$, then e_F is a soft δ −interior point of (U,E),there exists $(H_1,E) \in \tau_\delta$ such that $e_F \in (H_1,E) \sqsubseteq (U,E)$, Consequently $e_F \in \sqcup_{i\in I} \{(H_i,E),\} \in \tau_\delta, (H_i,E) \sqsubseteq (U,E)\}$. Therefore $Int_s^\delta(U,E) \sqsubseteq \sqcup_{i\in I} \{(H_i,E),\} \in \tau_\delta, (H_i,E) \sqsubseteq (U,E)\}$. We are done

(3) By (2) above $Int_s^\delta(U,E) = \sqcup_{i\in I} \{(H_i,E),\} \in \tau_\delta, (H_i,E) \sqsubseteq (U,E)\}$. Thus $Int_s^\delta(U,E)$ s is a soft δ −open set soft subset of (U,E). Now let $(H,E) \in \tau_\delta$ and $e_F \in (H,E)$ then $e_F \in (H,E) \sqsubseteq (U,E)$. Therefore $e_F$is a soft interior point of (U,E).Thus $e_F \in (H,E) \sqsubseteq Int_s^\delta(U,E)$ which shows that every soft δ −open soft subset of (U,E) is contained in $Int_s^\delta(U,E)$. Hence $Int_s^\delta(U,E)$ is the largest soft δ −open set contained in (U,E).

(4) Let (U,E) is soft δ −open set. Then a soft δ −open set containing all of its soft points, it follows that every soft point of (U,E) is a soft δ −interior of (U,E). Thus $Int_s^\delta(U,E) \sqsubseteq (U,E)$ Let $e_F \in Int_s^\delta(U,E)$ there exists $(H,E) \in \tau_\delta$ such that $e_F \in (H,E) \sqsubseteq (U,E)$ then $e_F \in$

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$Int_s^{\delta}(U,E) \sqsubseteq (U,E)$ Therefore $Int_s^{\delta}(U,E) = (U,E)$ Conversely, since $Int_s^{\delta}(U,E) = (U,E)$, then $(U,E) \in \tau_\delta$. Hence (U,E) is soft $\delta-$open set if and only if $Int_s^{\delta}(U,E) = (U,E)$.

(5) Is obvious.

(6) Since $Int_s^{\delta}(U,E) \in S\delta O(X,E)$ is soft $\delta-$open set. We have $Int_s^{\delta}\ Int_s^{\delta}(U,E) = Int_s^{\delta}(U,E)$

(7) Let $e_F \in Int_s^{\delta}(H,E)$ there exists $(U,E) \in \tau_\delta$ soft containing e_F such that

$(U,E) \sqsubseteq (H;E)$. But $(H,E) \sqsubseteq (G,E)$. Then $(U,E) \sqsubseteq (G;E)$. which implies that

$e_F \in Int_s^{\delta}(G,E)$. Thus $Int_s^{\delta}(H,E) \sqsubseteq Int_s^{\delta}(G,E)$

(8) Since $(H,E) \sqsubseteq (H,E) \sqcup (G,E)$. We have $Int_s^{\delta}(H,E) \sqsubseteq Int_s^{\delta}((H,E) \sqcup (G,E))$ and $(H,E) \sqsubseteq (H,E) \sqcup (G,E)$. Then $Int_s^{\delta}(G,E) \sqsubseteq Int_s^{\delta}((H,E) \sqcup (G,E))$ Therefore $Int_s^{\delta}((H,E) \sqcup (G,E)) = Int_s^{\delta}(H,E) \sqcup Int_s^{\delta}(G,E)$.

(9) Since $(H,E) \sqcap (G,E) \sqsubseteq (H;E)$. Then $Int_s^{\delta}((H,E) \sqcap (G,E)) \sqsubseteq Int_s^{\delta}(H;E)$

Also $Int_s^{\delta}((H,E) \sqcap (G,E)) \sqsubseteq Int_s^{\delta}(G;E)$

We get $Int_s^{\delta}((H,E) \sqcap (G,E)) \sqsubseteq Int_s^{\delta}(H;E) \sqcap Int_s^{\delta}(G;E)$. Conversely, let $e_F \in Int_s^{\delta}(H;E) \sqcap Int_s^{\delta}(G;E)$. So that $e_F \in Int_s^{\delta}(H;E)$and $e_F \in Int_s^{\delta}(G;E)$. There exists two soft $\delta-$open set $(U_1,E),(U_2,E)$ soft containing e_F such that $(U_1,E) \sqsubseteq (H,E)$, and $(U_2,E) \sqsubseteq (G,E)$ Implies that $(U_1,E) \sqcap (U_2,E) = (U,E) \in S\delta O(X,E)$, and $e_F \in (U,E) \sqsubseteq (H,E) \sqcap (G,E)$. (U;E) Then $e_F \in Int_s^{\delta}((H,E) \sqcap (G,E))$. We get $Int_s^{\delta}(H,E) \sqcap Int_s^{\delta}(G,E) \sqsubseteq Int_s^{\delta}((H,E) \sqcap (G,E))$. Therefor $Int_s^{\delta}(H,E) \sqcap Int_s^{\delta}(G,E) = Int_s^{\delta}((H,E) \sqcap (G,E))$. The following example shows that the equalities do not hold in Proposition 4.14(8).

Example 4.15. The soft topological space $(X,\tau,E)$is the same as in Example 3.2:

Suppose that $(U,E) = \{(e_1,\{h_1\}),(e_2,\{h_1,h_3\})\}$ and $(G,E) = \{(e_1,\{h_2,h_3\}),(e_2,\{h_1,h_2\})\}$. One can deduce that $Int_s^{\delta}(H,E) \sqcup Int_s^{\delta}(G,E) = Int_s^{\delta}((H,E) \sqcup (G,E))$., and $Int_s^{\delta}(H,E) \sqcap Int_s^{\delta}(G,E) \neq Int_s^{\delta}((H,E) \sqcap (G,E))$.

Corollary 4.16. Let (X,τ_δ,E) be a soft topological space, and $(U,E) \in SS(X,E)$. Then

$$Int_s^{\delta}(U,E) = \sqcup \{(H_i,E) : (H_i,E) \sqsubseteq (U,E), (H_i,E) \in SRO(X,E)\}.$$

Definition 4.17. Let (X,τ,E) be a soft topological space and $e_F \in SP(X)$ is said to be a soft $\delta-$cluster point of $(F,E) \in SS(X,E)$ if for every a soft regular open (U,E) soft containing of e_F we have $(F,E) \sqcap (U;E) \neq 0_E$. The set of all soft $\delta-$cluster points of (G,E) is called the soft $\delta-$closure denoted by $Cl_s^{\delta}(G,E)$

Remark 4.18. Let (X,τ,E)be a soft topological space and $(F,E) \in (X;E)$, then.

$$Cl_s^{\delta}(F,E) = \sqcap \{(H_i,E) : (F,E) \sqsubseteq (H_i,E), (H_i,E) \in SRC(X,E)\}.$$

Proposition 4.19. Let(X,τ,E) be a soft topological space over X and $(V,E),(F,E) \in SS(X,E)$. Then

(1) $Cl_s^{\delta}(V,E) \in S\delta C(X,E)$.

(2) $(V,E) \sqsubseteq Cl_s^{\delta}(V,E)$.

(3) (V,E) is soft $\delta-$closed set if and only if $Cl_s^{\delta}(V,E) = (V,E)$.

(4) $Cl_s^{\delta}(F,E) = \sqcap \{(H_i,E) \in S\delta C(X,E), (F,E) \sqsubseteq (H_i,E)\}$.

(5) $Cl_s^{\delta}(0_E) = 0_E$ and $Cl_s^{\delta}(1_E) = 1_E$.

(6) $Cl_s(V,E)) \sqsubseteq Cl_s^{\delta}(V,E)$.

(7) $Cl_s^{\delta}Cl_s^{\delta}(F,E) = Cl_s^{\delta}(F,E)$

(8) If $(V,E) \sqsubseteq (F,E)$, then $Cl_s^{\delta}(V,E) \sqsubseteq Cl_s^{\delta}(F,E)$.

(9) $Cl_s^{\delta}(V,E) \sqcup Cl_s^{\delta}(G,E) = Cl_s^{\delta}((V,E) \sqcup (G,E))$.

(10) $Cl_s^{\delta}((H,E) \sqcap (V,E)) \sqsubseteq Cl_s^{\delta}(H,E) \sqcap Cl_s^{\delta}(V,E)$.

The following example shows that the equalities do not hold in Proposition 4.19(9)

Example 4.20. The soft topological space (X,τ,E) is the same as in Example 3.2. Suppose $(H,E) = \{(e_1,\{h_2\}),(e_2,\{h_2\})\}$ and $(M,E) = \{(e_1,\{h_1\}),(e_2,\{h_3\})\}$ $Cl_s^{\delta}(H,E) = \{(e_1,\{h_2,h_3\}),(e_2,\{h_2,h_3\})\}$, $Cl_s^{\delta}(|M,E) = \{(e_1,\{h_1,h_3\}),(e_2,\{h_1,h_3\})\}$, $Cl_s^{\delta}((H,E) \sqcap (M,E)) = Cl_s^{\delta}(0_E) = 0_E$. Therefore $Cl_s^{\delta}(H,E) \sqcap Cl_s^{\delta}(M,E) \not\sqsubseteq Cl_s^{\delta}((H,E) \sqcap (M,E))$

Proposition 4.21. Let (X,τ,E) be a soft topological space, and $(G;E) \in SS(X,E)$. Then.

(1) $Cl_s^\delta(V^c,E) = (Int_s^\delta(V,E))^c$

(2) $(Cl_s^\delta(V,E))^c = Int_s^\delta(V^c,E)$.

(3) $Int_s^\delta(V,E) = (Cl_s^\delta(V^c,E))^c$

Proof. (1) We prove that) $Cl_s^\delta(V^c,E)) \sqsubseteq (Int_s^\delta(V,E))^c$ Let $e_F \in Cl_s^\delta(V^c,E)$. Then, for every soft regular open set (U,E) containing of e_F, we have $(V^c,E) \sqcap (U,E) \neq 0_E$. Then the relation $(U,E) \sqsubseteq (V,E)$ is not true. Therefore $e_F \notin Int_s^\delta(V,E)$. Then $e_F \in (Int_s^\delta(V,E))^c$ thus $Cl_s^\delta(V^c,E) \sqsubseteq (Int_s^\delta(V,E))^c$

Conversely, we prove that $Int_s^\delta(V,E))^c \sqsubseteq Cl_s^\delta(V^c,E)$. Let $e_F \in (Int_s^\delta(V,E))^c$ and $e_F \notin Int_s^\delta(V,E)$. There (U,E) be a soft regular open set containing of e_F . such that $e_F \in (U,E) \sqsubseteq (V^c,E)$ therefore have $(V^c,E) \sqcap (U,E) \neq 0_E$. Thus, $e_F \in Cl_s^\delta(V^c,E)$.

We gets $Int_s^\delta(V,E))^c \sqsubseteq Cl_s^\delta(V^c,E)$. Therefore $l_s^\delta(V^c,E) = (Int_s^\delta(V,E))^c$.

(2) $(Cl_s^\delta(V,E))^c = (\sqcap\{(H_i,E) \in S\delta C(X,E), (H_i,E) \sqsubseteq (V,E)\})^c = \sqcup\{(H_i^c,E) \in S\delta O(X,E), (V^c,E) \sqsubseteq (H_i^c,E)\} = Int_s^\delta(V^c,E)$

(3) Obvious.

3 Soft δ −Boundary and Soft δ −Exterior

Definition 5.1. Let (X,τ,E) be a soft topological space and (F,E) be a soft set over X. The soft δ −boundary of soft set (F,E) over X is denoted by $Bd_s^\delta(F,E)$ and is defined as $Bd_s^\delta(F,E) = Cl_s^\delta(F,E) \sqcap Cl_s^\delta(F^c,E)$.

Remark 5.2. From the above definition it follows directly that the soft sets (F,E) and (F^c,E) have same soft δ −boundary.

Proposition 5.3. Let (X,τ,E) be a soft topological space, $(F,E) \in SS(X,E)$. Then the following statements are true.

(1) $Bd_s^\delta(F,E) = Cl_s^\delta(F,E) \setminus Int_s^\delta(F,E)$.

(2) $Bd_s^\delta(F,E) \sqcap Int_s^\delta(F,E) = 0_E$.

(3) $(F,E) \sqcup Bd_s^\delta(F,E) = Cl_s^\delta(F,E)$

Proof. (1) $Cl_s^\delta(F,E) \setminus Int_s^\delta(F,E) = Cl_s^\delta(F,E) \sqcap (Int_s^\delta(F,E))^c = Cl_s^\delta(F,E) \sqcap Cl_s^\delta(F^c,E) = Bd_s^\delta(F,E)$.

(2)) $Bd_s^\delta(F,E) \sqcap Int_s^\delta(F,E) = Cl_s^\delta(F,E) \sqcap Cl_s^\delta(F^c,E) \sqcap Int_s^\delta(F,E) = Cl_s^\delta(F,E) \sqcap Cl_s^\delta(F^c,E) \sqcap (Cl_s^\delta(F,E))^c = Cl_s^\delta(F,E) \sqcap (Cl_s^\delta(F^c,E) \sqcap (Cl_s^\delta(F,E))^c) = Cl_s^\delta(F,E) \sqcap 0_E = 0_E$.

(3) $((F,E) \sqcup Bd_s^\delta(F,E) = (F,E) \sqcup Cl_s^\delta(F,E) \sqcap Cl_s^\delta(F^c,E) = \left((F,E) \sqcup Cl_s^\delta(F,E)\right) \sqcap (F,E) \sqcup Cl_s^\delta(F^c,E) = Cl_s^\delta(F,E) \sqcap ((F,E) \sqcup Cl_s^\delta(F^c,E) \sqsupseteq Cl_s^\delta(F,E) \sqcap \left((F,E) \sqcup (F^c,E)\right)) = Cl_s^\delta(F,E) \sqcap 1_E = Cl_s^\delta(F,E)$.

Theorem 5.4. Let (X,τ,E) be a soft topological space, $(F,E) \in SS(X,E)$. Then $Bd_s^\delta(F,E) = 0_E$ if and only if (F,E) is soft δ −closed set and soft δ −open set .

Proof. Suppose that $Bd_s^\delta(F,E) = 0_E$ First, we prove that (F,E) is soft δ −closed set. We have $Bd_s^\delta(F,E) = 0_E$ or $Cl_s^\delta(F,E) \sqcap Cl_s^\delta(F^c,E) = 0_E$ Hence $Cl_s^\delta(F,E) \sqsubseteq (Cl_s^\delta(F^c,E))^c = Int_s^\delta(F,E)$. Therefore $Cl_s^\delta(F,E) \sqsubseteq (F,E)$, and so (F,E) is soft δ −closed set. Now, we prove that (F,E) is soft δ −open set. Indeed, we have $Bd_s^\delta(F,E) = 0_E$ or $Cl_s^\delta(F,E) \sqcap Cl_s^\delta(F^c,E) = 0_E$. Hence $(F,E) \sqcap (Int_s^\delta(F,E))^c = 0_E$, and so $(F,E) \sqsubseteq Int_s^\delta(F,E)$. Therefore (F,E) is soft δ −open set. Conversely, suppose that (F,E) is soft δ −closed set and soft δ −open set. Then $Bd_s^\delta(F,E) = Cl_s^\delta(F,E) \sqcap Cl_s^\delta(F^c,E) = Cl_s^\delta(F,E) \sqcap Int_s^\delta(F,E) = (F,E) \sqcap (F,E) = 0_E$

Theorem 5.5. Let (X,τ,E) be a soft topological space, $(F,E) \in SS(X,E)$, then

(1) (F,E) is soft δ −open set if and only if $(F,E) \sqcap Bd_s^\delta(F,E) = 0_E$

(2) $((F,E)$ is soft δ − closed set if and only if $Bd_s^\delta(F,E) \sqsubseteq (F,E)$

Proof. (1) Let (F,E) is soft δ −open set. Then $Int_s^\delta(F,E) = (F,E)$. Implies that $(F,E) \sqcap Bd_s^\delta(F,E) = 0_E$ Conversely, suppose that $(F,E) \sqcap Bd_s^\delta(F,E) = 0_E$Then $(F,E) \sqcap (Cl_s^\delta(F,E) \sqcap$

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$Cl_s^\delta(F^c,E)) = 0_E$ or

Then $((F,E) \sqcap Cl_s^\delta(F,E)) \sqcap Cl_s^\delta(F^c,E) = 0_E$Then $(F,E) \sqcap Cl_s^\delta(F,E) = 0_E$ which implies that $(F,E) \sqsubseteq (Cl_s^\delta(F,E))^c = Int_s^\delta(F,E)$. Moreover, $Int_s^\delta(F,E) \sqsubseteq (F,E)$. Therefore $Int_s^\delta(F,E) = (F,E)$. And thus (F,E) is soft δ −open set.

(2) Let (F,E) be a soft $\delta -$ closed set . Then $Cl_s^\delta(F,E) = (F,E)$. Now, $Bd_s^\delta(F,E) = Cl_s^\delta(F,E) \sqcap Cl_s^\delta(F^c,E) \sqsubseteq (F,E)$. That is, $Bd_s^\delta(F,E) \sqsubseteq (F,E)$. Conversely, suppose that $Bd_s^\delta(F,E) \sqsubseteq (F,E)$. Then $Bd_s^\delta(F,E) \sqcap (F^c,E) = 0_E$. Since $d_s^\delta(F,E) = Bd_s^\delta(F^c,E)$, we have Then $Bd_s^\delta(F^c,E) \sqcap (F^c,E) = 0_E$. By (1), (F^c,E) is soft δ −open set. Hence (F,E) is soft δ −closed set.

Proposition 5.6. Let (X,τ,E) be a soft topological space, $(F,E) \in SS(X,E)$. Then the following statements are true.

(1) $(F,E) \setminus Bd_s^\delta(F,E) = Int_s^\delta(F,E)$.

2) If(F,E) is soft δ −closed set, then $(F,E) \setminus Int_s^\delta(F,E) = Bd_s^\delta(F,E)$

Proof. (1)We have $(F,E) \setminus Bd_s^\delta(F,E) = (F,E) \sqcap (Bd_s^\delta(F,E))^c = (F,E) \sqcap ((Cl_s^\delta(F,E) \sqcap Cl_s^\delta(F^c,E))^c)$

$= (F,E) \sqcap ((Cl_s^\delta(F,E) \sqcap (Int_s^\delta(F,E))^c)^c)$

$= (F,E) \sqcap (Cl_s^\delta(F,E))^c \sqcup \left((F,E) \sqcap Int_s^\delta(F,E)\right)$

$= 0_E \sqcup Int_s^\delta(F,E) = Int_s^\delta(F,E)$

(2) Since (F,E) is soft δ −closed set, $Cl_s^\delta(F,E) = (F,E)$. Therefore $(F,E) \setminus Int_s^\delta(F,E) = Cl_s^\delta(F,E) \setminus Int_s^\delta(F,E) = Bd_s^\delta(F,E)$

Definition 5.7. Let (X,τ,E) be a soft topological space and $(F,E) \in SS(X,E)$. The δ −exterior of (F,A) is the soft set. $Ext_s^\delta(F,E) = Int_s^\delta(F^c,E)$ is said to be a soft δ −exterior of (F,A).

Theorem 5.8. Let (X,τ,E) be a soft topological space and $(F,E),(G,E) \in SS(X,E)$. Then the following statements hold.

(1) $Ext_s^\delta(F,E) \sqsubseteq Ext(F,E)$ is soft δ −open set.

(2) $Ext_s^\delta(F,E)$ is soft δ −open set.

(3) $Ext_s^\delta(F,E) = 0_E \setminus Cl_s^\delta(F,E)$.

(4) $Ext_s^\delta\left(Ext_s^\delta(F,E)\right) = Int_s^\delta\left(Cl_s^\delta(F,E)\right)$.

(5) If $(F,E) \sqsubseteq (G,E)$, then $Ext_s^\delta(F,E) \sqsupseteq Ext_s^\delta(G,E)$.

(6) $Ext_s^\delta\big((F,E) \sqcup (G,E)\big) = Ext_s^\delta(F,E) \sqcap Ext_s^\delta(G,E)$.

(7) $Ext_s^\delta(F,E) \sqcap Ext_s^\delta(G,E) \sqsubseteq Ext_s^\delta((F,E) \sqcap (G,E))$

(8) $Ext_s^\delta(0_E) = 0_E$, and $Ext_s^\delta(1_E) = 1_E$

(9) $Ext_s^\delta\big((Ext_s^\delta(F,E))^c\big) = Ext_s^\delta(F,E)$

(10) $Ext_s^\delta(F,E) \sqcup Ext_s^\delta(F,E) \sqsubseteq Ext_s^\delta((F,E) \sqcap (G,E))$ (11) $Int_s^\delta(F,E) \sqsubseteq Ext_s^\delta\left(Ext_s^\delta(F,E)\right)$.

Proof. (1) $Ext_s^\delta(F,E) = Int_s^\delta(F^c,E) \sqsubseteq Int_s(F^c,E) = Ext_s(F,E)$.

(2) Since $Int_s^\delta(F,E)$ is soft δ −open set Then $Ext_s^\delta(F,E)$ is soft δ −open set.

(3) Obvious

(4) $Ext_s^\delta\left(Ext_s^\delta(F,E)\right) = Int_s^\delta(Ext_s^\delta(F,E))^c = Int_s^\delta(Int_s^\delta(F^c,E))^c = Int_s^\delta\left(Cl_s^\delta(F,E)\right)$.

(5) If $(F,E) \sqsubseteq (G,E)$ then $(G^c,E) \sqsubseteq (F^c,E)$ and $Int_s^\delta(F^c,E) \sqsupseteq Int_s^\delta(G^c,E)$. Thus $Ext_s^\delta(F,E) \sqsupseteq Ext_s^\delta(G,E)$

(6) $Ext_s^\delta\big((F,E) \sqcup (G,E)\big) = Int_s^\delta((F,E) \sqcup (G,E))^c = Int_s^\delta\big((F^c,E) \sqcap (G^c,E)\big) = Int_s^\delta(F^c,E) \sqcap Int_s^\delta(G^c,E) = Ext_s^\delta(F,E) \sqcap Ext_s^\delta(G,E)$

(7) $Ext_s^\delta(F,E) \sqcap Ext_s^\delta(G,E) = Int_s^\delta(F^c,E) \sqcap Int_s^\delta(G^c,E) \sqsubseteq Int_s^\delta\big((F^c,E) \sqcap (G^c,E)\big) = Int_s^\delta((F,E) \sqcap (G,E))^c$

$= Ext_s^\delta\big((F,E) \sqcup (G,E)\big) \sqsubseteq Ext_s^\delta((F,E) \sqcap (G,E))$

(8) ,(9),(10) are obvious.

(11) $Int_s^\delta(F,E) \sqsubseteq Int_s^\delta\left(Cl_s^\delta(F,E)\right) = Int_s^\delta(Int_s^\delta(F^c,E))^c$.

$= Int_s^\delta(Ext_s^\delta(F,E))^c = Ext_s^\delta(Ext_s^\delta(F,E))$.

The following example shows that the equalities do not hold in Proposition 5.8 (7),(10) and (11).

Example 5.9. In Example 3.2 (1), we obtain $Ext_s^\delta(F,E) \sqcap Ext_s^\delta(G,E) \sqsubseteq Ext_s^\delta((F,E) \sqcap (G,E))$ and $Ext_s^\delta(F,E) \sqcap Ext_s^\delta(G,E) \neq Ext_s^\delta((F,E) \sqcap (G,E)$. Similarly, we find That (2) $Ext_s^\delta(F,E) \sqcup Ext_s^\delta(F,E) \sqsubseteq Ext_s^\delta\big((F,E) \sqcap (G,E)\big)$, but $Ext_s^\delta(F,E) \sqcup Ext_s^\delta(F,E) \neq Ext_s^\delta\big((F,E) \sqcap$

$(G,E))$. In Example 3.2 (3), we have $Int_s^{\delta}(F,E) \sqsubseteq Ext_s^{\delta}\left(Ext_s^{\delta}(F,E)\right)$.

4 Soft $f_{Pu} - \delta$ −continuity

In this section, we introduce the notion of soft $f_{Pu} - \delta$ −continuity of functions induced by two mappings $u: X \longrightarrow Y$ and $p: E \longrightarrow B$ on soft topological spaces (X,τ,E) and (Y,τ^*,B)

Definition 6.1. Let (X,τ,E) and (Y,τ^*,B) be two soft topological spaces, $u: X \longrightarrow Y$ and $p: E \longrightarrow B$ be mappings, and $e_F \in SP(X)$. The map $f_{Pu}\colon \mathrm{SS(X,E)} \longrightarrow \mathrm{SS(Y,B)}$ is soft $f_{Pu} - \delta$ −continuous at e_F if for each (G,B) neighborhood e_F, there exists a (H,E) neighborhood e_F such that $f_{Pu}(Int_s(Cl_s(H,E)) \sqsubseteq (Int_s(Cl_s(G,B))$.

Theorem 6.2. For the mapping $f_{Pu}\colon \mathrm{SS(X,E)} \longrightarrow \mathrm{SS(Y,B)}$, the following properties are equivalent.

(1) f_{Pu} is soft δ −continuous.

(2) For each $e_F \in SP(X)$. and each soft regular open set (V,B) containing $f_{Pu}(e_F)$, there exists a soft regular open set (U,E) containing e_F such that $f_{Pu}(U,E) \sqsubseteq (V,E)$.

(3) $f_{Pu}Cl_s^{\delta}(A,E) \sqsubseteq Cl_s^{\delta}f_{Pu}(A,E)$ for every $(A,E) \in SS(X,E)$.

(4) $Cl_s^{\delta}f_{pu}^{-1}(N,B) \sqsubseteq f_{pu}^{-1}Cl_s^{\delta}(N,B)$ for every $(N,B) \in SS(Y,B)$.

(5) For each soft δ −closed set $(V,B) \in S\delta C(Y,B)$, $f_{pu}^{-1}(V,B) \in S\delta C(X,E)$.

(6) For each soft δ −open set $(V,B) \in S\delta O(Y,B)$, $f_{pu}^{-1}(V,B) \in S\delta O(X,E)$.

(7) For each soft regular open set $(V,B) \in SS(Y,B)$, $f_{pu}^{-1}(V,B) \in S\delta O(X,E)$.

(8) For each soft regular closed set set $(N,B) \in SS(Y,B)$, $f_{pu}^{-1}(N,B) \in S\delta C(X,E)$.

Proof. (1) ⇒ (2): Directly from Definition 6.1.

(2) ⇒ (3): Let $e_F \in SP(X)$ and $(A,E) \in SS(X,E)$ such that $f_{Pu}(e_F) \in f_{Pu}Cl_s^{\delta}(A,E)$. Suppose that $f_{Pu}(e_F) \neq f_{Pu}Cl_s^{\delta}(A,E)$. Then, there exists a soft regular open set neighborhood (V,B) of $f_{Pu}(e_F)$ such that $f_{Pu}(A,E) \sqcap (V,B) = 0_B$. By (2), there exists a soft regular open set neighborhood (U,E) of (e_F) such that $f_{Pu}(U,E) \sqsubseteq (V,E)$. Since $f_{Pu}(A,E) \sqcap f_{Pu}(U,E) \sqsubseteq f_{Pu}(A,E) \sqcap (V,B) = 0_B$, $f_{Pu}(A,E) \sqcap f_{Pu}(U,E)$. Hence, we get that $(A,E) \sqcap (U,E) \sqsubseteq f_{pu}^{-1}(f_{Pu}(A,E)) \sqcap f_{pu}^{-1}(f_{Pu}(U,E)) = f_{pu}^{-1}(f_{Pu}(A,E) \sqcap f_{Pu}(U,E)) = 0_B$. Hence we have $(U,E) \sqcap (A,E) = 0_E$, and $(e_F) \notin Cl_s^{\delta}(A,E)$. This shows that $f_{Pu}(e_F) \in f_{Pu}Cl_s^{\delta}(A,E)$. This is a contradiction.

Therefore, we obtain that $f_{Pu}(e_F) \in Cl_s^{\delta}(f_{Pu}(A,E))$

(3) ⇒ (4): Let $(N,B) \in SS(Y,B)$ such that $(A,E) = f_{pu}^{-1}(N,B)$. By (3), $f_{Pu}\left(Cl_s^{\delta}\left(f_{pu}^{-1}(N,B)\right)\right) \sqsubseteq Cl_s^{\delta}\left(f_{Pu}\left(f_{pu}^{-1}(N,B)\right)\right) \sqsubseteq Cl_s^{\delta}(N,B)$. From here, we have $Cl_s^{\delta}\left(f_{pu}^{-1}(N,B)\right) \sqsubseteq f_{pu}^{-1}(Cl_s^{\delta}\left(f_{Pu}\left(f_{pu}^{-1}(N,B)\right)\right)) \sqsubseteq f_{pu}^{-1}(Cl_s^{\delta}(N,B))$. Thus we obtain $Cl_s^{\delta}\left(f_{pu}^{-1}(N,B)\right) \sqsubseteq f_{pu}^{-1}(Cl_s^{\delta}(N,B))$.

(4) ⇒ (5): Let $(V,B) \in S\delta C(Y,B)$. By (4), $Cl_s^{\delta}\left(f_{pu}^{-1}(V,B)\right) \sqsubseteq f_{pu}^{-1}\left(Cl_s^{\delta}(V,B)\right) = f_{pu}^{-1}(V,B)$. and ever $f_{pu}^{-1}(V,B) \sqsubseteq Cl_s^{\delta}\left(f_{pu}^{-1}(V,B)\right)$. Hence that $Cl_s^{\delta}\left(f_{pu}^{-1}(V,B)\right) = f_{pu}^{-1}(V,B)$. This shows that $f_{pu}^{-1}(V,B) \in S\delta C(X,E)$.

(5) ⇒ (6): Let $(V,B) \in S\delta O(Y,B)$. Then $(V^c,B) \in S\delta C(Y,B)$. By (5), $f_{pu}^{-1}(V^c,B) = \left(f_{pu}^{-1}(V,B)\right)^c \in S\delta C(Y,B)$. Therefore, $f_{pu}^{-1}(V^c,B) \in S\delta C(Y,B)$.

(6) ⇒ (7): Let $(V,B) \in SRO(Y,B)$. Since every a soft regular open set is soft δ −open set, $(V,B) \in S\delta O(Y,B)$. By (6), $f_{pu}^{-1}(V,B) \in S\delta O(X,E)$

(7) ⇒ (8): Let $(N,B) \in SRC(Y,B)$. Then $(N^c,B) \in SRO(Y,B)$. By (7), $f_{pu}^{-1}(N^c,B) = \left(f_{pu}^{-1}(N,B)\right)^c \in S\delta O(X,E)$. Therefore $f_{pu}^{-1}(N,B) \in S\delta C(X,E)$.

(8) ⇒ (1): Let $e_F \in SP(X)$. and $(A,B) \in SRO(Y,B)$ such that $f_{Pu}(e_F) \in (A,E)$. Now, then $(A^c,B) \in$

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$SRC(Y,B)$. By (8), $f_{pu}^{-1}(A^c,B) = \left(f_{pu}^{-1}(A,B)\right)^c \in S\delta C(X,E)$. Thus, $f_{pu}^{-1}(A,B) \in S\delta O(X,E)$. Since $(e_F) \in f_{pu}^{-1}(A,B)$. Then there exists $(U,E) \in SRO(Y,B)$ such that $e_F \in (U,E) \sqsubseteq f_{pu}^{-1}(A,B)$. Hence $f_{Pu}(Int_s(Cl_s(U,E)) \sqsubseteq (Int_s(Cl_s(A,B))$. This shows that f_{Pu} is a soft δ −continuous mapping.

Theorem 6.3. For the mapping f_{Pu}: SS(X, E) ⟶ SS(Y, B), the following properties are equivalent.

(1) f_{Pu} is soft δ −continuous.

(2) $f_{pu}^{-1} Int_s^{\delta}(U,B) \sqsubseteq Int_s^{\delta} f_{pu}^{-1}(U,B)$ for every $(U,B) \in SS(Y,B)$.

Proof. (1) ⇒ (2):Let f_{Pu} is soft δ −continuous and $(U,B) \in SS(Y,B)$. Implies that

$Cl_s^{\delta} f_{pu}^{-1}(U^c,B) \sqsubseteq f_{pu}^{-1} Cl_s^{\delta}(U^c,B)$. Then

$f_{pu}^{-1} Int_s^{\delta}(U,B) = f_{pu}^{-1}(Cl_s^{\delta}(U^c,B))^c =$

$(f_{pu}^{-1}(Cl_s^{\delta}(U^c,B)))^c \sqsubseteq (Cl_s^{\delta}\left(f_{pu}^{-1}(U^c,B)\right))^c =$

$Cl_s^{\delta}(f_{pu}^{-1}(U^c,B))^c = Int_s^{\delta} f_{pu}^{-1}(U,B)$.

(2) ⇒ (1): We prove that $f_{Pu}\,(U,B) \in S\delta O(X,E)$ for every (U,B) 2 $S\delta O(Y,B)$. Let $(U,B) \in S\delta O(Y,B)$, then $Int_s^{\delta}(U,B) = (U,B)$. Therefore, $f_{pu}^{-1}\,(U,B) = f_{pu}^{-1}(\,Int_s^{\delta}(U,B)) \sqsubseteq Int_s^{\delta}(f_{pu}^{-1}(U,B))$. We have $Int_s^{\delta}(f_{pu}^{-1}(U,B)) \sqsubseteq f_{pu}^{-1}(U,B)$. Thus $Int_s^{\delta}\left(f_{pu}^{-1}(U,B)\right) = f_{pu}^{-1}(U,B)$, which means that $f_{pu}^{-1}(U,B) \in S\delta O(X,E)$. Implies that f_{Pu} is soft δ −continuous.

5. Conclusion

All over the globe, soft set theory is a topic of interest for many authors working in diverse areas due to its rich potential for applications in several directions. So, we found it reasonable to extend some known concept in general topology to the soft topological structures. In this paper, several characterizations of soft δ −topology in terms of soft δ −open sets are introduced and the concept of soft $f_{Pu} - \delta$ −continuity is obtained. Thus we fill a gap in the existing literature on soft topology.

6. References

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