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FEWMA and Fuzzy Regression Model Control Chart one a-cut With Application

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ABSTRACT

In many real-world applications, the data to be used in a control charting method are not crisp since they are approximated due to environmental uncertainties, In these situations, fuzzy numbers and linguistic variables are used to grab such uncertainties. That is why the use of a fuzzy control chart, in which fuzzy data are used, is justified. As an exponentially weighted moving average (EWMA) scheme is usually used to detect small shifts, in this paper a fuzzy EWMA (F-EWMA) control chart is proposed to detect small shifts in the process mean when fuzzy data are available. As well as The fuzzy regression control chart which is a functional technique to evaluate the process in which the average has a trend and the data represents a linguistic or approximate value.

In this paper, the application of fuzzy logic in statistical quality control have been done by plotting fuzzy EWMA chart and Fuzzy linear regression model control chart depending on a suggested algorithm prepared for this purpose, as well as the theoretical structure of the "a-level fuzzy midrange for a-cut fuzzy -regression control chart" is proposed for triangular membership functions and applying that to the chemical analysis data of a water component, Total Dissolved Solid (TDS) in water from the KANY Factory to detect small shifts in the process means, the data contains three groups (TDS-a, TDS-b, TDS-c), for 24 days each day containing 5 hour data, as shown in Table (1)

The comparison showed that the fuzzy linear regression model Control chart is a good technique and more suitable, accuracy, sensitivity than the traditional linear regression model Control chart.

KEY WORDS: Fuzzy Sets, Fuzzy Numbers, Control Charts, Regression Control Chart, FEWMA, and Fuzzy Linear Regression model control charts.

1. Introduction

Statistical Process Control (SPC) is approaching that uses statistical techniques to monitor the process. The techniques of quality control are widely used in controlling any kinds of processes. The W.Shewhart 1924 is famous India statistics, he is the first one who produced the control chart. These are called a traditional variable control chart, which consists of three horizontal lines called Centre Line (CL), Upper Control Limit (UCL) and Lower Control Limit (UCL) are represented by numeric values. From the Shewhart chart (UCL, and LCL) defined as this formula:

$$\mathrm{UCL, LCL} = \mu_0 \mp d\sigma$$

Let be μ_w and σ_w be a mean and standard deviation of w sample statistics and d is the distance of the control limits from the center. And the Range chart:

$$\mathrm{UCL, LCL} = \bar{X} \mp A_2\bar{R}$$

Where $\bar{X}$ Is mean, A_2 is a constant tabulated and R is the average of Rang.

$$\mathrm{UCL} = D_4\bar{R}$$

$$\mathrm{LCL} = D_3\bar{R} \qquad \mathrm{CL} = \bar{R}$$

The D_3,and D_4 are the constant tabulated for R-Chart.[3]

The Exponentially Weighted Moving Average (EWMA) Control Chart was introduced by Roberts(1959) for monitoring the process mean is defined as:

$$Z_i = \lambda X_i + (1-\lambda)Z_{i-1} \quad \text{Where} \quad Z_0 = \mu_0$$

$$\mathrm{UCL, LCL} = \mu_0 \pm L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}[1-(1-\lambda)^{2i}]} \quad \ldots(1)$$

Note that the equation $[1-(1-\lambda)^{2i}]$ become unity as i gets larger. Then the UCL, LCL of EWMA it becomes:

$$\mathrm{UCL, LCL} = \mu_0 \pm L\sigma\sqrt{\frac{\lambda}{(2-\lambda)}} \quad \ldots(2)$$

The fuzzy theory was first proposed in 1965 by Zadeh.

It is a mathematical tool that deals with uncertainty which comes from a shortage of information, incompleteness, vagueness and inaccurate of measurements. The fuzzy EWMA control chart was proposed by Şenturk et al, Suggested a fuzzy EWMA (F-EWMA) control chart to study fuzzy process data using α-cuts, we used FEWMA control chart for monitoring the Yarn process.

Fuzzy EWMA statistics and their calculations are defined and the proposed method for drawing and analyzing control charts based on these statistics are discussed. Fuzzy EWMA statistics are used in order to monitor each one of the profile parameters. The Fuzzy Exponential Weighted Moving average FEWMA Control chart when $\sigma_a, \sigma_b, \sigma_c$ are known, there for $\bar{R}_a, \bar{R}_b, \bar{R}_c$ Are average of the range, where the range of (a, b, c) obtained as followings:

$$R_{aj} = Max(X_{aj}) - Min(X_{cj})$$
$$R_{bj} = Max(X_{bj}) - Min(X_{bj})$$
$$R_{cj} = Max(X_{cj}) - Min(X_{aj}) \quad ....(3)$$

Where the $max(X_{ij})$, and $Min(X_{ij})$ are a fuzzy numbers in the sample.

When detecting a small shift in process with fuzzy observations, the FEWMA control chart should be used to evaluate the process, $\bar{X}_{a1}, \bar{X}_{b1}, \bar{X}_{c1}$ Are the fuzzy average of sample size n. The Fuzzy exponentially weighted moving average is defined as: .[2][3][4][5][8][9]

$$\begin{aligned} Z_1 &= \lambda(\bar{X}_{a1}, \bar{X}_{b1}, \bar{X}_{c1}) + (1-\lambda)(\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c) \\ Z_2 &= \lambda(\bar{X}_{a1}, \bar{X}_{b1}, \bar{X}_{c1}) + (1-\lambda)Z_1 \\ Z_3 &= \lambda(\bar{X}_{a2}, \bar{X}_{b2}, \bar{X}_{c2}) + (1-\lambda)Z_2 \quad ....(4) \\ &\vdots \\ Z_m &= \lambda(\bar{X}_{am}, \bar{X}_{bm}, \bar{X}_{cm}) + (1-\lambda)Z_{m-1} \end{aligned}$$

$$UCL, LCL_{EWMA} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c)\frac{3}{\sqrt{n}}(\sigma_a, \sigma_b, \sigma_c)\sqrt{\frac{1}{2-\lambda}}$$

$$UCL, LCL_{EWMA} = \bar{\bar{X}}_a \mp \frac{3\sigma_a}{\sqrt{n}}\sqrt{\frac{1}{2-\lambda}}, \bar{\bar{X}}_b \mp \frac{3\sigma_b}{\sqrt{n}}\sqrt{\frac{1}{2-\lambda}}, \bar{\bar{X}}$$

$$CL_{EWMA} = (\bar{\bar{X}}_a, \bar{\bar{X}}_b, \bar{\bar{X}}_c)$$

Where the standard deviations and fuzzy averages are knowing.

The α-level fuzzy median for α-cuts FEWMA control chart for $(\sigma_a, \sigma_b, \sigma_c)$ are known, and the transformation techniques are applied to α-cuts FEWMA control charts for obtaining the crisp value of control limits, the α-level fuzzy median for α-cuts are obtained for the sample number t is moderately large and t is small as follows:

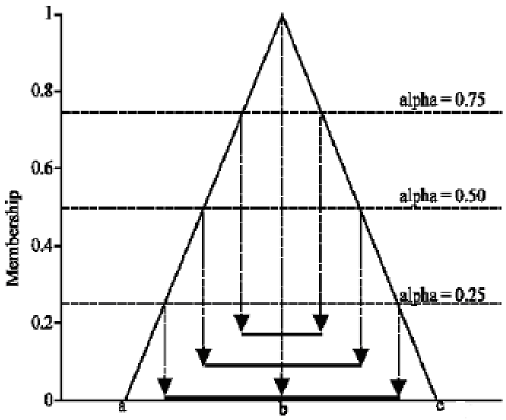
$$UCL^{\alpha}_{med-EWMA}, LCL^{\alpha}_{med-EWMA} = CL^{\alpha}_{med-EWMA} \mp \frac{1}{\sqrt{n}}(\sigma^{\alpha}_a, \sigma_b, \sigma^{\alpha}_c)\sqrt{\frac{\lambda}{2-\lambda}} \quad ....(7)$$
$$CL^{\alpha}_{med-EWMA} = \frac{1}{3}(\bar{\bar{X}}^{\alpha}_a, \bar{\bar{X}}_b, \bar{\bar{X}}^{\alpha}_c)$$

The Sample with FEWMA control chart, we can calculate the α-level fuzzy median value if the standard deviation is unknown then to calculate the FEWMA is this formula **[3][4][7][8]**

$$= \bar{\bar{X}}_a \mp A_2\bar{R}_a\sqrt{\frac{\lambda}{2-\lambda}}, \bar{\bar{X}}_b \mp A_2\bar{R}_b\sqrt{\frac{\lambda}{2-\lambda}}, \bar{\bar{X}}_a \mp A_2\bar{R}_c\sqrt{\frac{\lambda}{2-\lambda}} \quad ....(8)$$
$$S^{\alpha}_{med-EWMA} = \frac{1}{3}(\bar{X}^{\alpha}_{aj}, \bar{X}^{\alpha}_{bj}, \bar{X}^{\alpha}_{cj})$$

To define or monitoring the Process of the FEWMA where α =0.65 if the process for X_a, X_b, X_c then :

$$\begin{aligned} \bar{X}^{\alpha}_{aj} &= \bar{X}_{aj} + \alpha(\bar{X}_{bj} - \bar{X}_{aj}) \\ \bar{X}^{\alpha}_{bj} &= \bar{X}_{bj} + \alpha(\bar{X}_{aj} - \bar{X}_{bj}) \\ \bar{X}^{\alpha}_{cj} &= \bar{X}_{cj} + \alpha(\bar{X}_{cj} - \bar{X}_{bj}) \\ \bar{R}^{\alpha}_{aj} &= \bar{R}_{aj} + \alpha(\bar{R}_{bj} - \bar{R}_{aj}) \\ \bar{R}^{\alpha}_{bj} &= \bar{R}_{bj} + \alpha(\bar{R}_{aj} - \bar{R}_{bj}) \\ \bar{R}^{\alpha}_{cj} &= \bar{R}_{cj} + \alpha(\bar{R}_{cj} - \bar{R}_{bj}) \end{aligned} \quad j = 1, \dots, n \quad ....(9)$$



Fig(1) Triangular fuzzy parameter and its α -cuts.

After calculating the fuzzy number with α-cuts for average for the first sample α-level fuzzy median value is obtained. Since this value is between control limits, the first sample is in control. Also the condition of process control for each sample is defined by using the following: [10][11]

$$Process\ control = \begin{cases} in-control, for\ LCL^{\propto}_{med-EWMA} < S^{\propto}_{med-EWMA} < UCL^{\propto}_{med-EWMA} \\ out-of\ control, \qquad otherwise \end{cases}$$

Where:

$$S^{\propto}_{med-EWMA.j} = \frac{1}{3}\left(\bar{X}^{\propto}_{aj} + \bar{X}^{\propto}_{bj} + \bar{X}^{\propto}_{cj}\right) \quad ....(10)$$

2. Fuzzy –Regression Control Chart:[6][7][11][13]

Liner model is one of the most efficient statistical tools that are used in many ways, Using regression techniques on the experimental observations has allowed the study of many phenomena in various fields of science as: Agriculture, Chemistry, Medicine, Environ UCL_{EWMA}, LCL_{EWMA}ent, Psychology, Biology, and Economics the regression model is of the statistical approach to study the relation between variables to describe the status of data fit the model and to control it.

Fuzzy linear models describe linear relations between relationship between a dependent variable and a set of independent ones, the input data (independent variable X), the output data (dependent variable Y).

In data types of model parameters, as well as a number of the parameters. Since the model selection is primarily data type driven, we used the data type as the feature for classification of the models.

We estimate the following general fuzzy linear model as

$$y_i = \beta_0 + \beta_1 X_i + \varepsilon \qquad \text{where } i = 1, ..., n \qquad ... \ (11)$$

The parameter (β0, β1) is estimated by using the following normal equations, solving a linear squares problem

$$\hat{\beta}_0 = \frac{\sum_{j=1}^{m} \bar{X}_j - \hat{\beta}_1 \sum_{j=1}^{m} t_j}{m}$$
$$\hat{\beta}_1 = \frac{\sum_{j=1}^{m} \bar{X}_j \,(t_j - \bar{t})}{\sum_{j=1}^{m} (t_j - \bar{t})^2} \qquad ....(12)$$

$Where\ \ the\ value\ \ of\ \ \bar{t}\ is\ average\ t_j$

Where the $\bar{X}_j$ Represents the average of n observations in the j^{th} Sample, and the $\bar{\bar{X}}$ Is the average of all mean. The traditional regression control chart, there for the UCL, LCL is:

$$UCL, LCL_{,Reg-X_j} = \hat{\beta}_0 + \hat{\beta}_1 t_j \mp A_2 \bar{R}$$
$$CL_{Reg-X} = \hat{\beta}_0 + \hat{\beta}_1 t_j \qquad ....(13)$$

Where the A_2 is a constant value and the $\bar{R}$ is the average of the range.

3. Fuzzy X-bar regression control chart [1][9][14]

The data are represented as fuzzy numbers as $(X_{aij}, X_{bij}, X_{cij})$, and the average of fuzzy sample is $(\bar{X}_{aij}, \bar{X}_{bij}, \bar{X}_{cij})$, are calculated as follows:

$$\bar{X}_{a.ij} = \frac{\sum_{i=1}^{m} x_{a.ij}}{n}$$
$$\bar{X}_{b.ij} = \frac{\sum_{i=1}^{m} x_{b.ij}}{n} \qquad ....(14)$$
$$\bar{X}_{c.ij} = \frac{\sum_{i=1}^{m} x_{c.ij}}{n}$$

Where: $i = 1 .... n\ and\ j = 1 ... m$

The fuzzy linear regression model of each fuzzy number can be the following equations:

$$\bar{X}_{Reg-a.j} = \hat{\beta}_{0a} + \hat{\beta}_{1a} t_j + \varepsilon$$
$$\bar{X}_{Reg-b.j} = \hat{\beta}_{0b} + \hat{\beta}_{1b} t_j + \varepsilon \qquad ....(15)$$
$$\bar{X}_{Reg-c.j} = \hat{\beta}_{0c} + \hat{\beta}_{1c} t_j + \varepsilon$$

And β_{0c}, β_{1c} Are estimated with similar calculations for the $\bar{X}_{bj}$ and $\bar{X}_{cj}$

And we are calculating the

Where $\bar{X}_{a.j} = \bar{X}_{a.j} - \bar{\bar{X}}_a$, $j = 1 \ldots . m$

$$\bar{\bar{X}}_a = \frac{\sum_{j=1}^{m} \bar{X}_{a.j}}{m}$$
$$\bar{\bar{X}}_b = \frac{\sum_{j=1}^{m} \bar{X}_{b.j}}{m} \quad \ldots .(16)$$
$$\bar{\bar{X}}_c = \frac{\sum_{j=1}^{m} \bar{X}_{c.j}}{m}$$

The mean of range is:

$$\bar{R}_a = \frac{\sum_{j=1}^{m} R_{a.j}}{m}, \quad \bar{R}_b = \frac{\sum_{j=1}^{m} R_{b.j}}{m}, \quad \bar{R}_c = \frac{\sum_{j=1}^{m} R_{c.j}}{m} \quad \ldots .(17)$$

$$UCL, LCL_{,Reg-\bar{X}_j} = (\hat{\beta}_{a0} + \hat{\beta}_{1a}t_j \mp A_2\bar{R}_a, \hat{\beta}_{b0} + \hat{\beta}_{1b}t_j \mp A_2\bar{R}_b, \hat{\beta}_{c0} + \hat{\beta}_{1c}t_j \mp A_2\bar{R}_c)$$
$$CL_{Reg-\bar{X}_j} = ((\hat{\beta}_{a0} + \hat{\beta}_{1a}t_j, \hat{\beta}_{b0} + \hat{\beta}_{1b}t_j, \hat{\beta}_{c0} + \hat{\beta}_{1c}t_j) \quad \ldots .(18)$$

The α-cut fuzzy X-regression control chart for TFN , then the $\bar{X}^{\alpha}_{Reg-a.j}$, and $\bar{X}^{\alpha}_{Reg-c.j}$ are:

$$\bar{X}^{\alpha}_{Reg-a.j} = (1-\alpha)(\beta_{0a} - \beta_{1a} t_j) + \alpha(\beta_{0b} - \beta_{1b} t_j)$$
$$\bar{X}^{\alpha}_{Reg-c.j} = (1-\alpha)(\beta_{0c} - \beta_{1c} t_j) + \alpha(\beta_{0b} - \beta_{1b} t_j)$$
$$\bar{R}^{\alpha}_{Reg-a.j} = \bar{R}_a + \alpha(\bar{R}_b - \bar{R}_a)$$
$$\bar{R}^{\alpha}_{Reg-c.j} = \bar{R}_c + \alpha(\bar{R}_c - \bar{R}_b) \quad \ldots .(19)$$

The α- level fuzzy midrange for α-cut fuzz $\bar{\bar{X}}$-Regression control chart can be calculated by using the midrange. Transformation technique as follows:

$$UCL^{\alpha}_{mi-Reg-\bar{X}_j}, LCL^{\alpha}_{mi-Reg-\bar{X}_j} = \frac{(\bar{X}^{\alpha}_{Reg-a.j} + \bar{X}^{\alpha}_{Reg-c.j})}{2} \mp A_2\left(\frac{\bar{R}^{\alpha}_a + \bar{R}^{\alpha}_c}{2}\right) \quad \ldots (20)$$

$$CL^{\alpha}_{mi-Reg-\bar{X}_j} = \frac{(\bar{X}^{\alpha}_{Reg-a.j} + \bar{X}^{\alpha}_{Reg-c.j})}{2} \quad \ldots .(21)$$

$$LCL^{\alpha}_{med-Reg-\bar{X}_j} \le S^{\alpha}_{med-Reg-\bar{X}_j} \le UCL^{\alpha}_{med-Reg-\bar{X}_j} \quad \ldots .(22)$$

4. Application: Numerical illustration

In this paper is to analysis the using the fuzzy FEWMA and fuzzy $\bar{\bar{X}}$-Regression control chart for the chemical analysis data for one of the components of water is Total Dissolved Solid (TDS) from KANY factory, the data contain three groups(TDS-a, TDS-b, TDS-c) , the data for 24 days each days contain 5 hours, as shows in table (1).

Table (1): Total Dissolved Solid (TDS)in the water from KANY factory

Group 1	TDS-a	TDS-b	TDS-c	Range
1	116	120	115	5
2	114	126	120	12
3	127	119	120	8
4	124	120	120	4
5	120	124	120	4
Average	120.20	121.80	119.00	
Range	13	7	5	
Group 2				
Group 2	122	125	127	5
1	123	128	122	6
2	125	125	122	3
3	124	124	122	2
4	120	120	121	1
5	122	125	127	5
Average	122.80	124.40	122.80	
Range	5	8	6	
.	.	.	.	.
.	.	.	.	.
.	.	.	.	.
Group 24				
1	121	117	119	4
2	128	122	124	6
3	122	123	121	2
4	118	124	125	7
5	119	118	125	7
Average	121.60	120.80	122.80	
Range	10	7	6	
$\bar{\bar{X}}$	119.95	120.43	119.97	
$\bar{R}$	10.13	8.79	8.67	

Table (1) contains the sub average and range of (24) groups, although contain the all total, average of (24) groups and average of the range. By using the equation (4) and data in table (1) we can calculate the fuzzy exponentially weighted moving average for Group (X_a, X_b, X_c), as is shown in table (2)

Table (2): FEWMA value

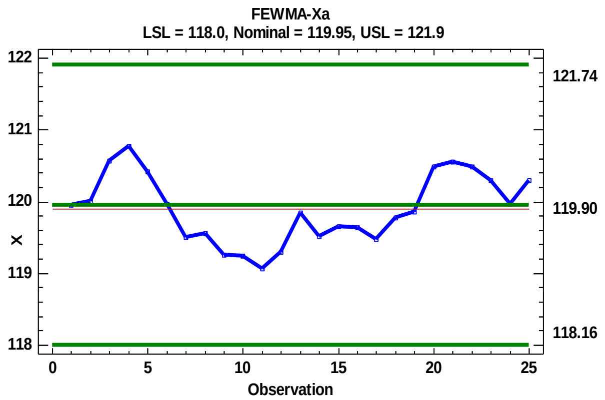
	Za	Zb	Zc
Z-0	119.95	120.43	119.97
Z-1	120.00	120.71	119.77
Z-2	120.56	121.45	120.38
Z-3	120.77	121.32	120.66
Z-4	120.41	120.89	120.13
Z-5	119.97	120.03	119.54
Z-6	119.50	119.75	119.40
Z-7	119.56	119.84	119.20
.	.	.	.
.	.	.	.
.	.	.	.
Z-20	120.55	120.94	121.09

Z-21	120.48	121.27	120.95
Z-22	120.30	120.74	120.44
Z-23	119.96	120.27	120.11
Z-24	120.29	120.38	120.65

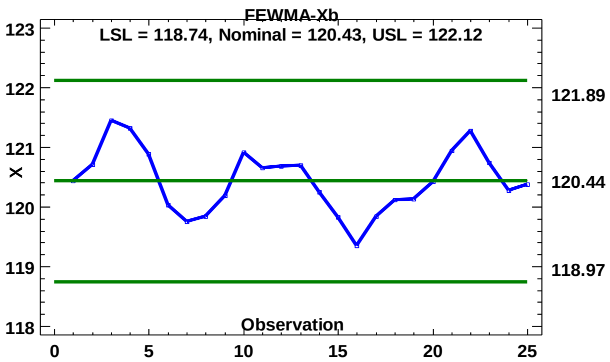
To define the fuzzy EWMA control chart are using the equation (6) and data Table (1) the value of UCL_{EWMA}, LCL_{EWMA} are (121.90, 118.0) and CL of FEWMA a is (119.95) as in table (3), and Fig(2), Fig(3), Fig(4) show that the distributed of the FEWMA control chart for X_a, X_b, X_c it is seen that there is no point out of control.

Table (3) UCL, LCL of FEWMA

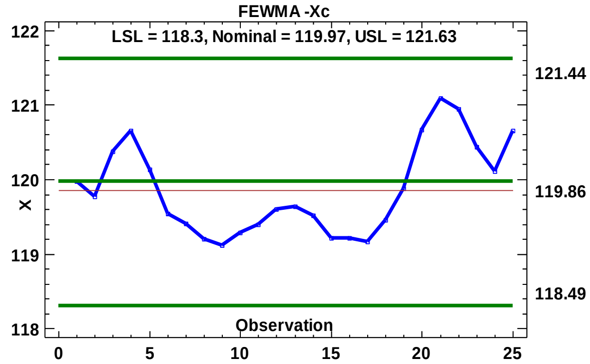
	Xa	Xb	Xc
UCL a	121.90	122.1243	121.6336
CL a	119.95	120.43	119.9667
LCL a	118.0026	118.7424	118.2998



Fig(2): FEWMA control chart of X_a



Fig(3): FEWMA control chart of X_b



Fig(4): FEWMA control chart of

The limit of the Fuzzy median transformation technique is integrated to the α-level fuzzy median calculated by the equation of (7) with value of α is (0.65) the value $\bar{\bar{X}}^{\alpha}_{a}$, $\bar{R}^{\alpha}_{a}$ as shows in table (4), calculated by equation (9), and the determined the $UCL^{\alpha}_{med-EWMA}$ and $LCL^{\alpha}_{med-EWMA}$ (121.8, 118.35) for sample size (24) by equation (8) as shows in table (4), and calculated the α-level fuzzy median value of $S^{\alpha}_{med-EWMA}$ For 24 samples at shows in table (5), from the table (6) shows that there are (6) from (24) points are out of control and as shows in Fig(5).

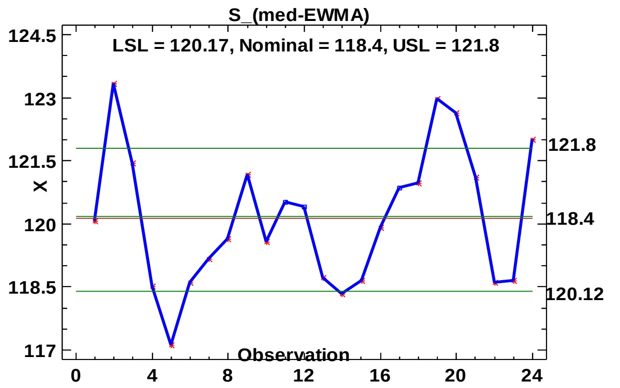
Table(4) evaluate $UCL^{\alpha}_{med-EWMA}$ and $LCL^{\alpha}_{med-EWMA}$

$\bar{\bar{X}}^{\alpha}_{a}$	**120.2642**	
$\bar{\bar{X}}^{\alpha}_{c}$	**120.27**	
$\bar{R}^{\alpha}_{a}$	9.258333	
$\bar{R}^{\alpha}_{c}$	8.747917	
α -Cut FEWMA		
FEWMA	**UCL**	**LCL**
X_a	**122.0449**	**118.4835**
X_b	**122.1243**	**118.7424**
X_c	**121.9525**	**118.5875**
Fuzzy median Transformation Tech. α -cut EWMA		
$UCL^{\alpha}_{med-EWMA}$	**121.8**	
$LCL^{\alpha}_{med-EWMA}$	**118.35**	

Table (5) α-level fuzzy median value of $S^{\alpha}_{med-EWMA}$

P	$\bar{X}^{\alpha}_{aj}$	$\bar{X}^{\alpha}_{bj}$	$\bar{X}^{\alpha}_{cj}$	$S^{\alpha}_{med-Reg-\bar{X}}$	121.8<S<118.35
1	121.24	121.80	117.18	120.0733	In control
2	123.84	124.40	121.76	123.3333	Out Of Control
3	121.08	120.80	122.45	121.4433	In control
4	119.13	119.20	117.22	118.5167	In control
5	117.16	116.60	117.59	117.1167	Out Of Control
6	118.25	118.60	118.93	118.5933	In control
7	120.06	120.20	117.23	119.1633	In control

8	120.34	121.60	116.98	119.64	In control
9	122.19	123.80	117.53	121.1733	In control
10	119.18	119.60	119.93	119.57	In control
11	120.59	120.80	120.14	120.51	In control
12	121.22	120.80	119.15	120.39	In control
13	118.33	118.40	119.39	118.7067	In control
14	118.9	118.20	117.87	118.3233	Out Of Control
15	118.17	117.40	120.37	118.6467	In control
16	120.75	121.80	117.18	119.91	In control
17	121.13	121.20	120.21	120.8467	In control
18	120.2	120.20	122.51	120.97	In control
19	122.09	121.60	125.23	122.9733	Out Of Control
20	122.23	123.00	122.67	122.6333	Out Of Control
21	121.76	122.60	118.97	121.11	In control
22	118.95	118.60	118.27	118.6067	In control
23	118.47	118.40	119.06	118.6433	In control
24	121.08	120.80	124.1	121.9933	Out Of Control



Fig(5) α-level FEWMA control chart

5. Regression

The regression equations of the application were calculated by data as in table (1) and by using the equations for the α-level fuzzy midrange for α-cut fuzzy $\ddot{X}$ -regression control chart, and the "α-level fuzzy midrange for α-cut fuzzy R control chart .

To calculate the fuzzy X-linear regression model for each fuzzy number ($\overline{X}_{aj}$, $\overline{X}_{bj}$, $\overline{X}_{cj}$) Can be estimated by equation(5), and to calculate the parameter (β_{0a}, β_{1a}) Of fuzzy linear regression by Equation (12) with data in table (1), where the dependent variable is the arithmetic mean and the time (t) are independent variables:

$$\overline{X}_{Reg-a.j}\ ,\ \overline{X}_{Reg-b.j}\ ,\ \overline{X}_{Reg-c.j}\ ,$$

Where:

$\overline{X}_{Reg-a.j} = 119.64 + 0.0289\ Ta$

$\overline{X}_{Reg-b.j} = 120.7 - 0.02\ Tb$

$\overline{X}_{Reg-c.j} = 119.2 + 0.07\ Tc$

To Estimate the α-cut fuzzy mean of X –regression $\overline{X}_{aj}^{0.65}$, $\overline{X}_{cj}^{0.65}$ by integrating an α-cut, where 0.65 By equation (19) we can calculate the α-cut fuzzy means of ranges $\overline{R}_{aj}^{0.65}$, $\overline{R}_{aj}^{0.65}$ As shows in table (6), and estimated the $UCL_{Regj}^{0.65}$ and $LCL_{Regj}^{0.65}$

Table (6) α-cut fuzzy mean of X –regression

$\overline{X}_{aj}^{0.65}$ = 120.3071 -0.0205 Tj	
$\overline{X}_{cj}^{0.65}$ = 120.1544 -0.035 Tj	
$\overline{R}_{aj}^{0.65}$ = 9.258	
$\overline{R}_{aj}^{0.65}$ = 8.748	
$UCL_{Regj}^{0.65}$	$LCL_{Regj}^{0.65}$
125.649+0.03Ta	114.965-0.0205Ta
125.74-0.02 Tb	115.595-0.02Tb
125.203+0.07 Tc	115.106+0.035Tc

α-cut fuzzy X –regression Control chart TFN by using the equation (19) was the $\overline{X}_{aj}^{0.65}$, $\overline{X}_{cj}^{0.65}$ value and the control chart as shows in table (7), if the α-cut, where 0.65 was selected are trend control. The process condition was set to evaluate the process with an the α-level fuzzy midrange transformation for the α-cut fuzzy X -regression and R control charts $UCL_{mi-Reg-\overline{X}_j}^{\alpha}$, $LCL_{mi-Reg-\overline{X}_j}^{\alpha}$. The Upper and lower calculated by the formula as equation(20) , where α=0.65 and A2=0.577 are given as shown in table (7).

Table (7)

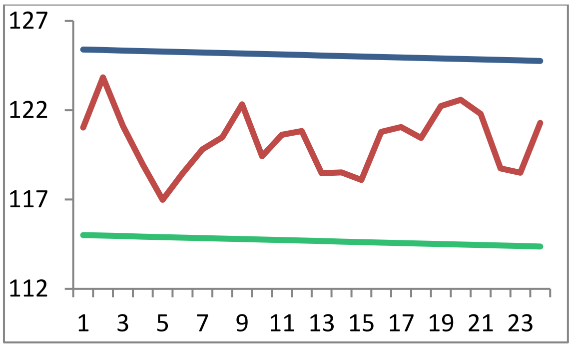
$UCL_{mi-Rg\,j}^{0.65}$ = 125.43 -0.028t
$CL_{mi-Rg\,j}^{0.65}$ = 120.23 -0.028t
$LCL_{mi-Rg\,j}^{0.65}$ = 115.- 0.028 T

The conditional process control limit of α-level fuzzy midrange for the α-cut fuzzy midrange transformation technique and the limit value of $S_{mi-\overline{X}_j}^{\alpha}$ Calculated for sample size (24) by using equation (20), the control limit is

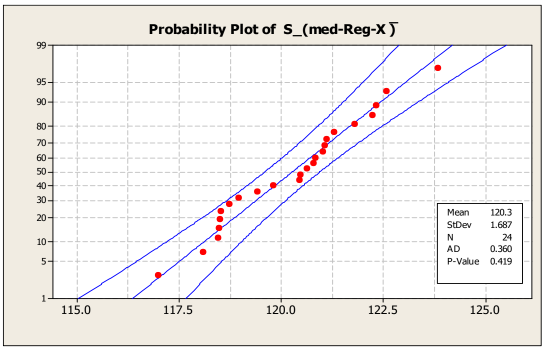
$$LCL_{med-Reg-\overline{X}_j}^{\alpha} \leq S_{med-Reg-\overline{X}_j}^{\alpha} \leq UCL_{med-Reg-\overline{X}_j}^{\alpha}$$

$$125.43 - 0.028\ T \leq S_{med-Reg-\overline{X}_j}^{\alpha} \leq 115\ - 0.028T$$

as shows in table (8)And the Fig(6) shows the probability plot of $S_{med-Reg-\overline{X}_j}^{\alpha}$, table (21) show that the process of control limits of the sample size (24) of TDS, it is seen that the all samples in control



Fig(6): Control Chart of $S^{\alpha}_{med-Reg-\bar{X}_j}$

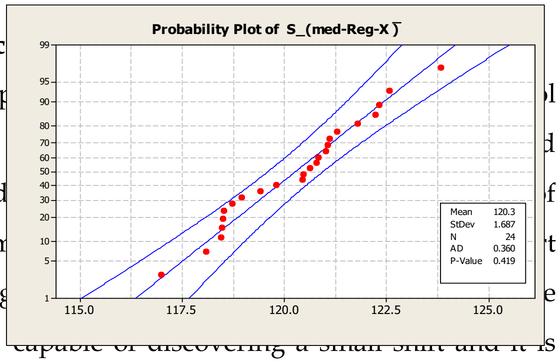


Fig(7)The probability plot of $S^{\alpha}_{med-Reg-\bar{X}_j}$

Table (8) Control limits using the ∝-level fuzzy midrange for the a-cut fuzzy $\widetilde{X}$ -regression control

	$UCL^{\alpha}_{med-Reg-\bar{X}}$	$S^{\alpha}_{med-Reg-\bar{X}}$	$LCL^{\alpha}_{med-Reg-\bar{X}}$	
	125.43-0.028 T		115 -0.028 T	
1	125.3977531	121.03	115.0081	Incontrol
2	125.3699749	123.84	114.9804	Incontrol
3	125.3421966	121.12	114.9526	Incontrol
4	125.3144183	118.96	114.9248	Incontrol
5	125.2866401	116.99	114.897	Incontrol
6	125.2588618	118.46	114.8693	Incontrol
7	125.2310836	119.82	114.8415	Incontrol
8	125.2033053	120.48	114.8137	Incontrol
9	125.175527	122.33	114.7859	Incontrol
10	125.1477488	119.43	114.7581	Incontrol
11	125.1199705	120.63	114.7304	Incontrol
12	125.0921923	120.84	114.7026	Incontrol
13	125.064414	118.47	114.6748	Incontrol
14	125.0366357	118.52	114.647	Incontrol
15	125.0088575	118.10	114.6193	Incontrol
16	124.9810792	120.79	114.5915	Incontrol
17	124.953301	121.06	114.5637	Incontrol
18	124.9255227	120.45	114.5359	Incontrol
19	124.8977444	122.23	114.5081	Incontrol
20	124.8699662	122.58	114.4804	Incontrol
21	124.8421879	121.80	114.4526	Incontrol
22	124.8144096	118.74	114.4248	Incontrol
23	124.7866314	118.51	114.397	Incontrol
24	124.7588531	121.29	114.3692	Incontrol

6. Conc



In this p... l charts ... d method ... f the com... t have a g... e highly capable of discovering a small shift and it is seen that fuzzy set theory is a good competent technique to draw the control chart. By comparing the two methods used to detect small deviations in the initial value of the mean or medium of the process, it was found that using FEWMA chart is more efficient than EWMA chart, when we have fuzzy numbers and linguistic variables used to grab such uncertainties, so it is necessary to use FEWMA Chart and diagnose deviations that may occur during the course of the production process. When the production process is in its beginner, it gives a clear picture of the production process through the curves of these charts.

And using the fuzzy - regression control chart to control the process of production and to solving the problem. In this paper used the fuzzy midrange transformation technique on the TDS data to transform data to crisp number. The numerical data on the α-level fuzzy midrange for the α-cut fuzzy – regression control chart idea to use the it is seen that the Fuzzy X- regression control chart is good technique and more suitable than a traditional regression control chart finally, it is seen the TDS data in control limit.

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